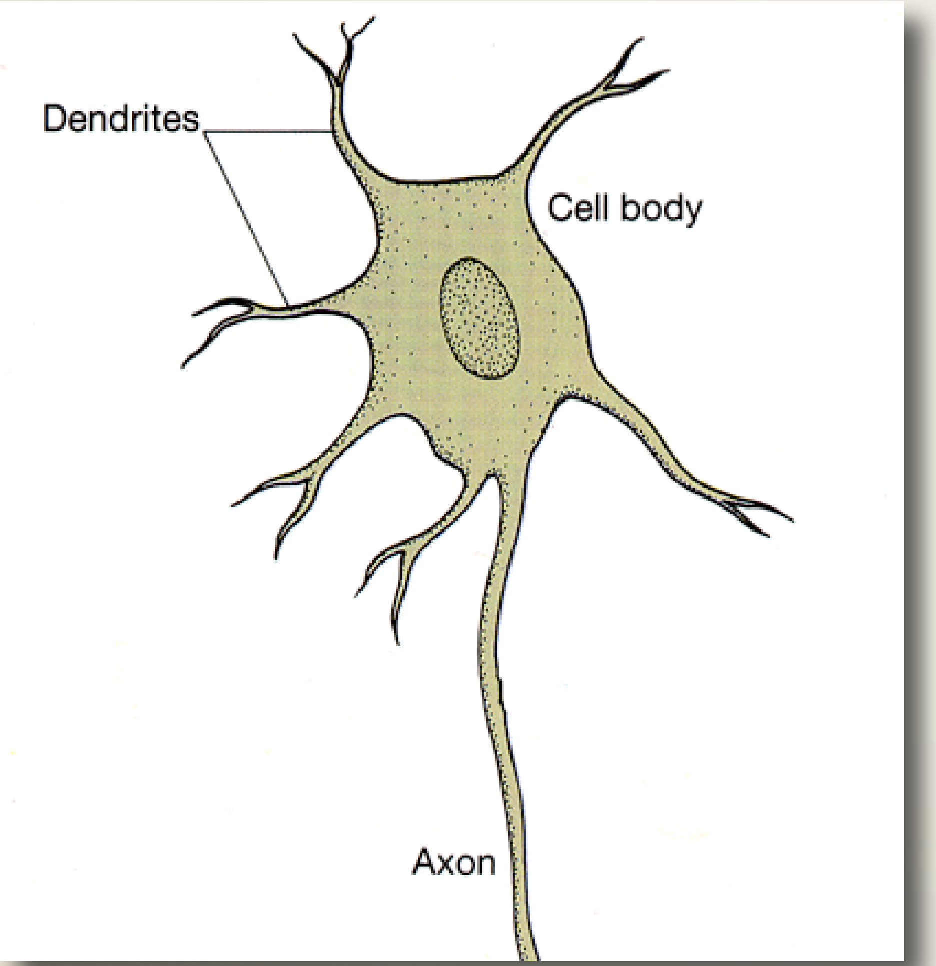
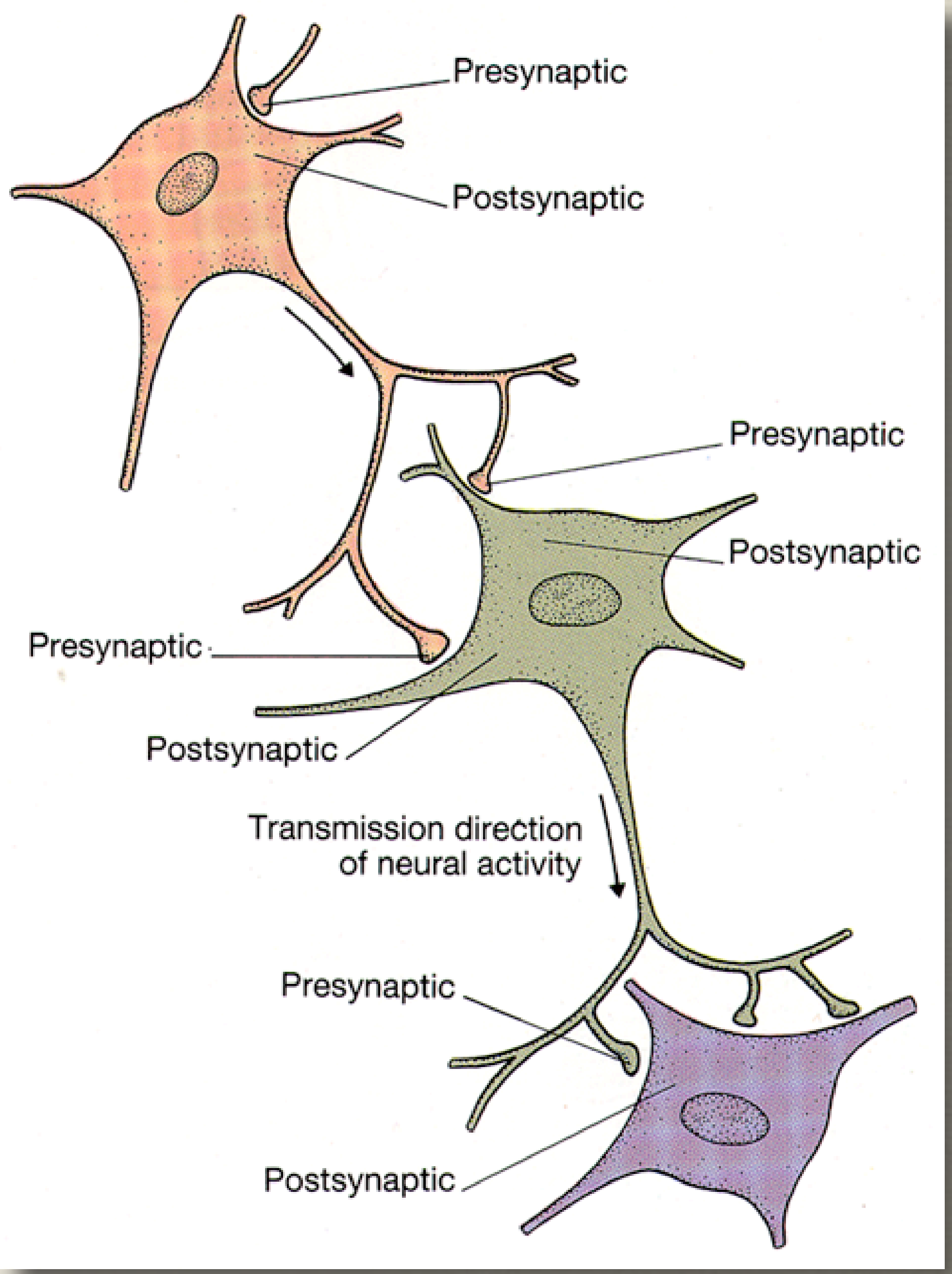
THE BRAIN

Neurons = Nerve Cells

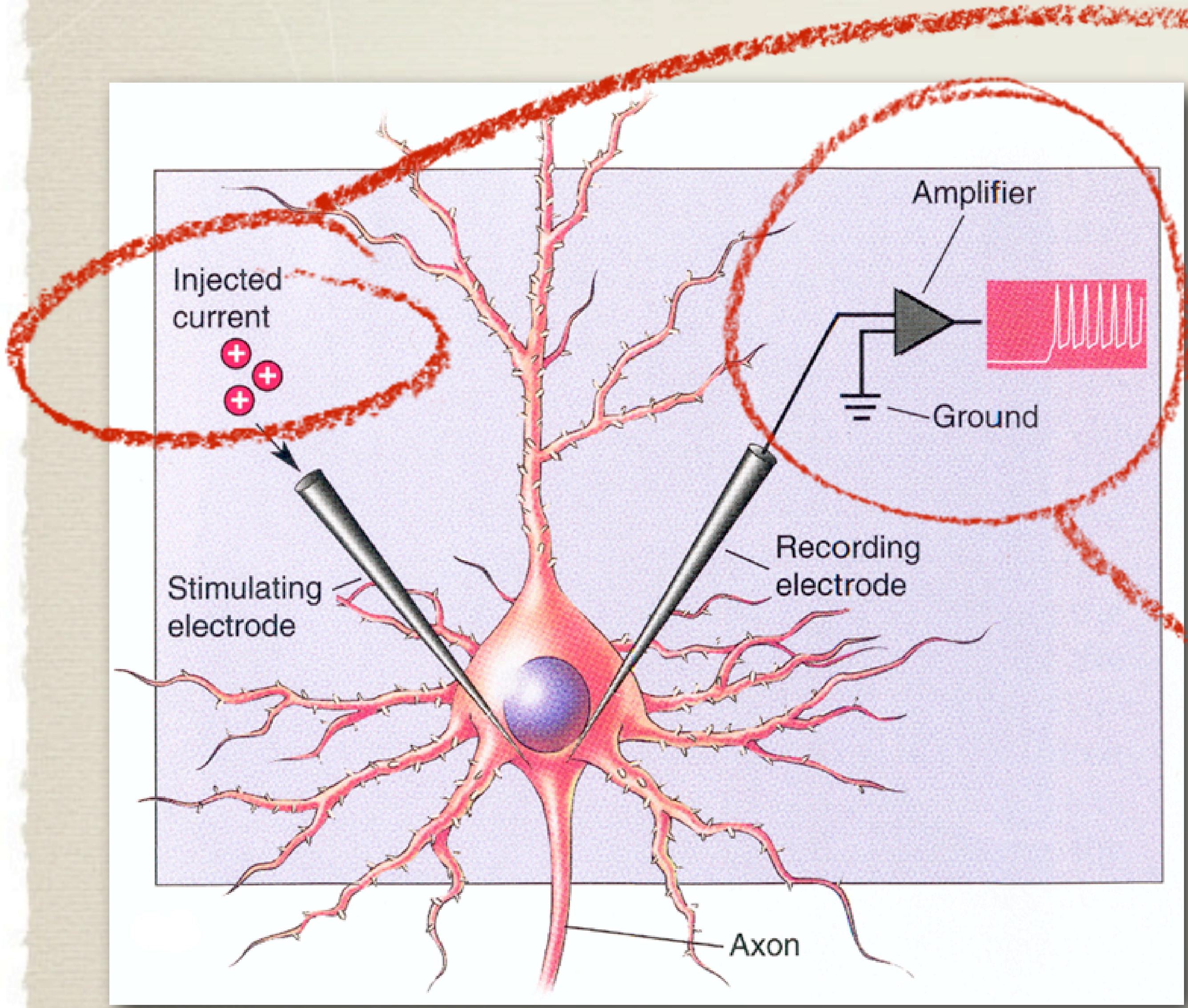
protoype neuron



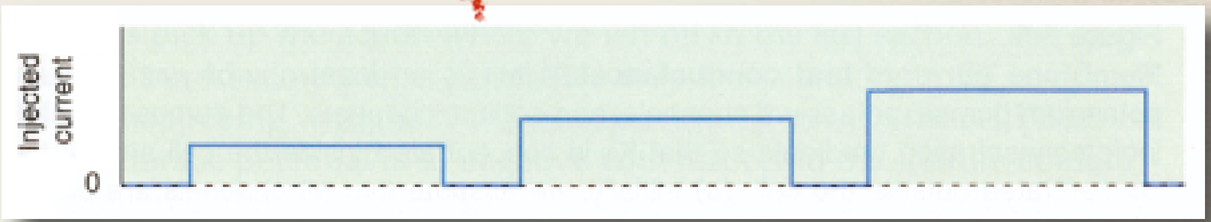
interconnected neurons



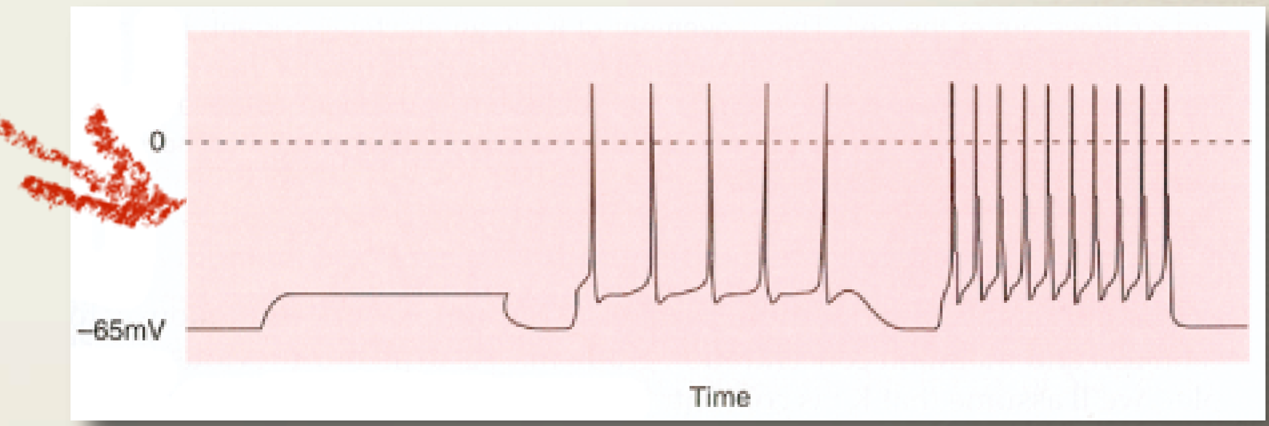
Inside The Neuron



Input



Output



Compare with Pulse Position Modulation

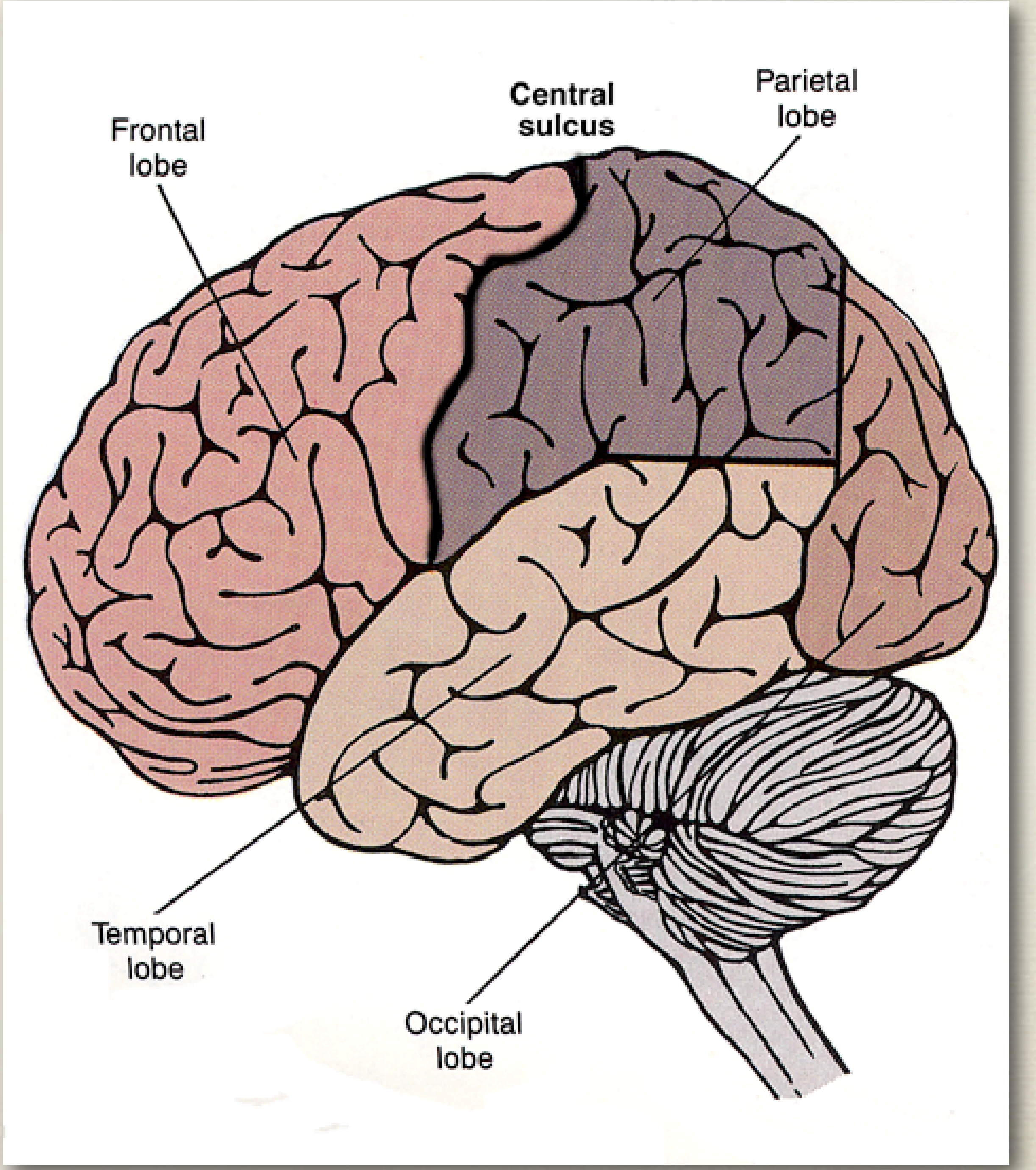
Cerebral Cortex and Its Lobes

Frontal lobe

Parietal lobe

Temporal lobe

Occipital lobe



The Cerebral Cortex: Some Basic Facts

* Organizes sensation, learning, movements, speech, and much more.
* 2–3 mm thick with about 10 billion neurons.
* Ridges & valleys make up for 2.5 m^2.
* Primary and secondary areas for information processing.

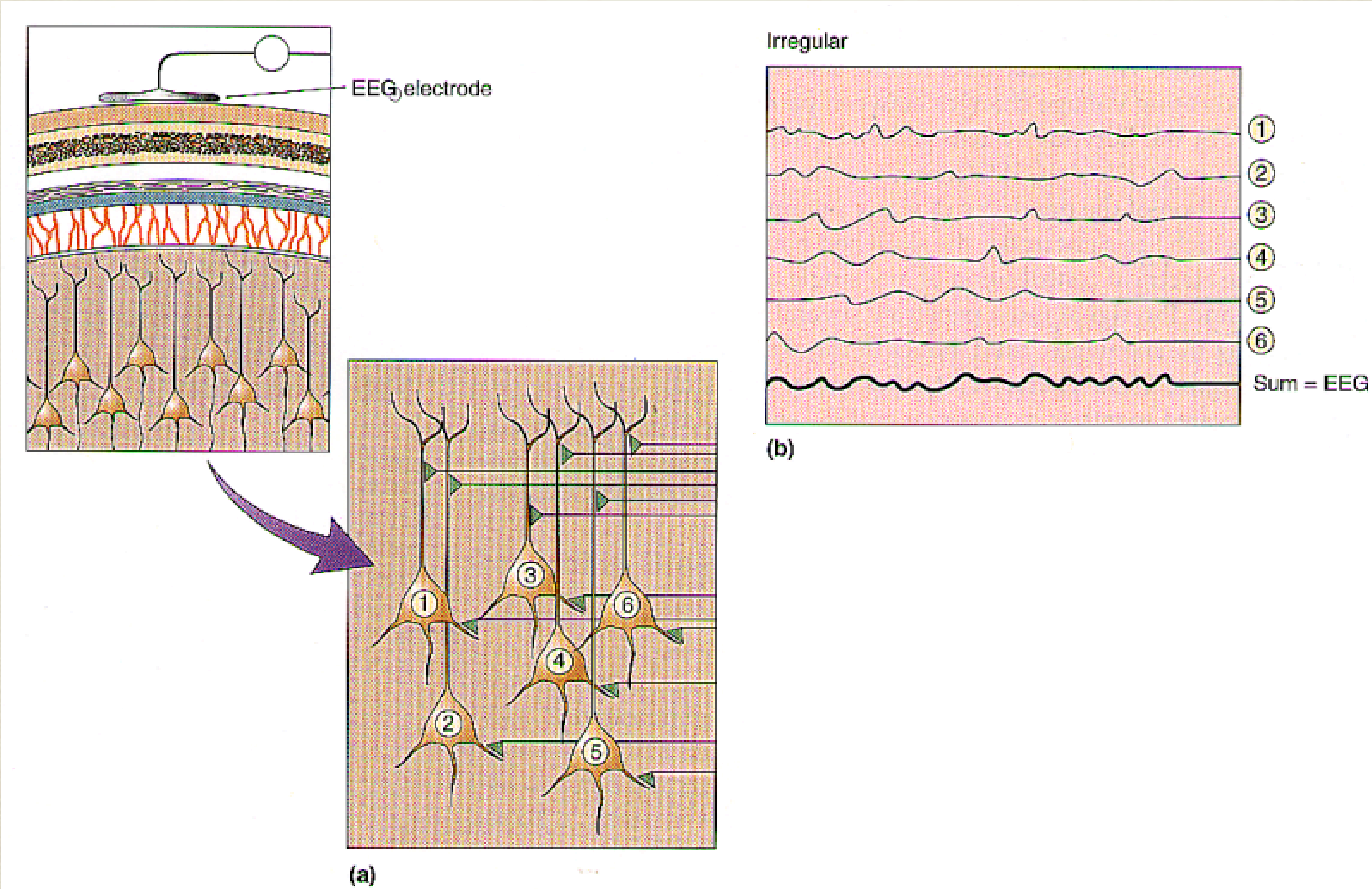
The Nervous System

* Somatic nervous system – controls, e.g., muscular activity in response to conscious commands ("foreign affairs").
* Autonomic nervous system – unconscious body control, e.g., of the heart rate ("internal affairs"),
    * sympathetic ("fight or flight")
    * parasympathetic ("read and digest")

Electroencephalogram – EEG

* The EEG reflects the electrical activity of the brain as measured on the scalp with several electrodes.
* Reflects the joint activity of millions of neurons.
* The EEG signal often has a rhythmic (oscillatory) pattern, characterized by frequency and amplitude.
* Different diseases are characterized by different rhythmic patterns.

EEG – (Un)synchronized Activity



EEG Acquisition

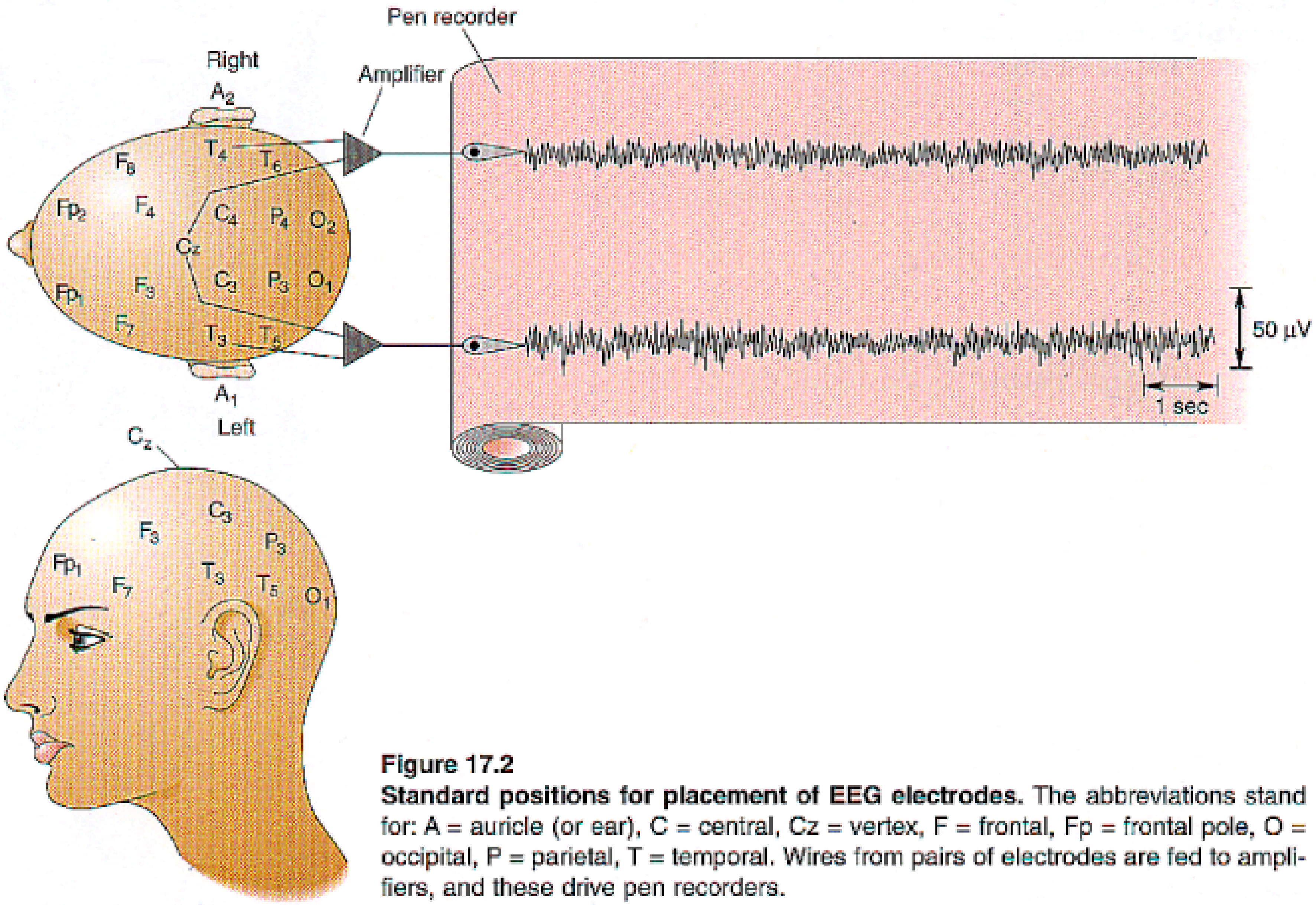


Figure 17.2
Standard positions for placement of EEG electrodes. The abbreviations stand for: A = auricle (or ear), C = central, Cz = vertex, F = frontal, Fp = frontal pole, O = occipital, P = parietal, T = temporal. Wires from pairs of electrodes are fed to amplifiers, and these drive pen recorders.

International 10/20 electrode system

Important EEG Rhythms

* Delta, <4 Hz: deep sleep, large amplitude
* Theta, 4–7 Hz: drowsy, sleep, pathological
* Alpha, 8–13 Hz: relaxed, awake, closed eyes (but suppressed when opened)
* Beta, 14–30 Hz: active cortex, low amplitude

EEG Signals – Examples

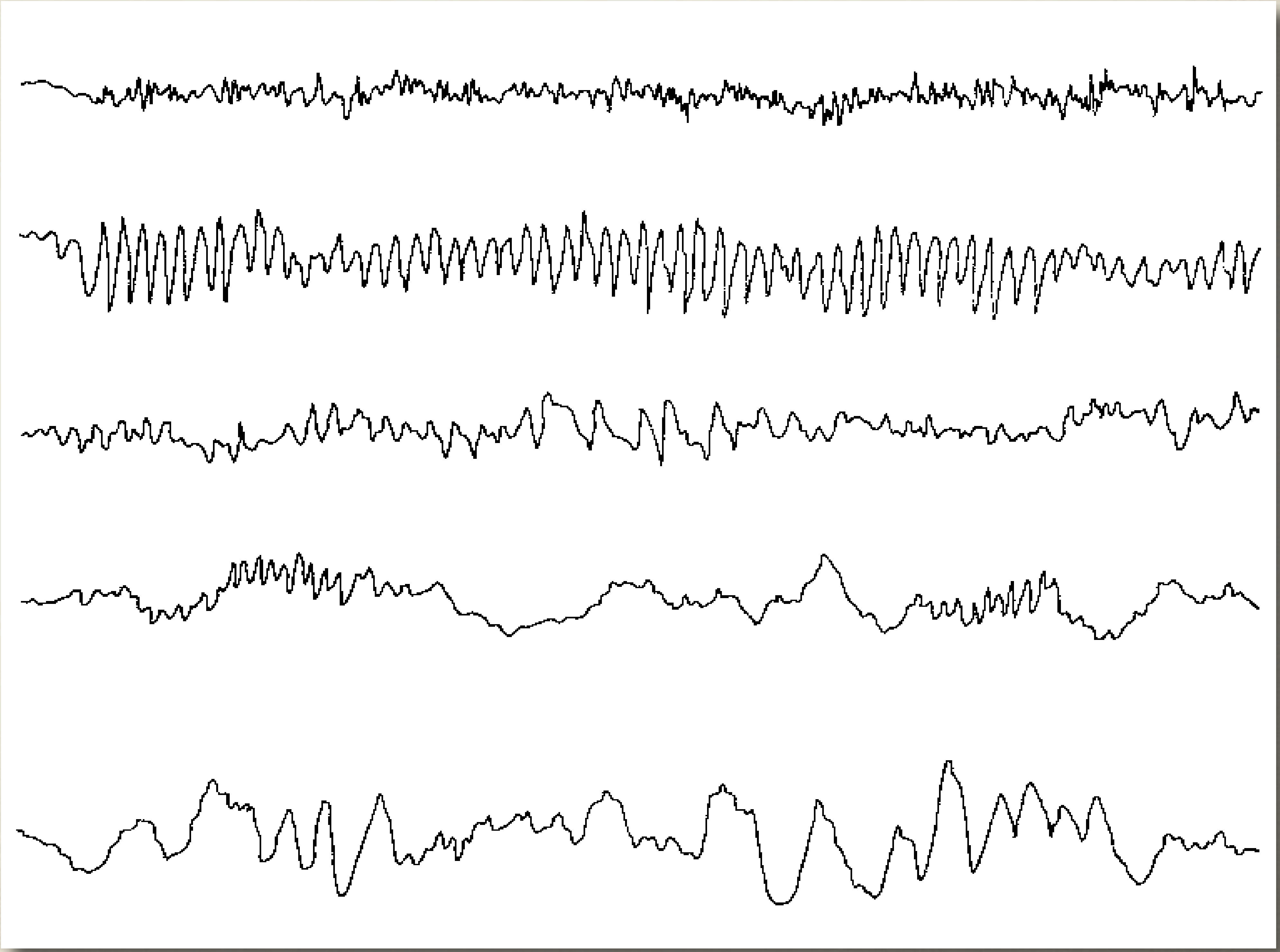
Active

Relaxed

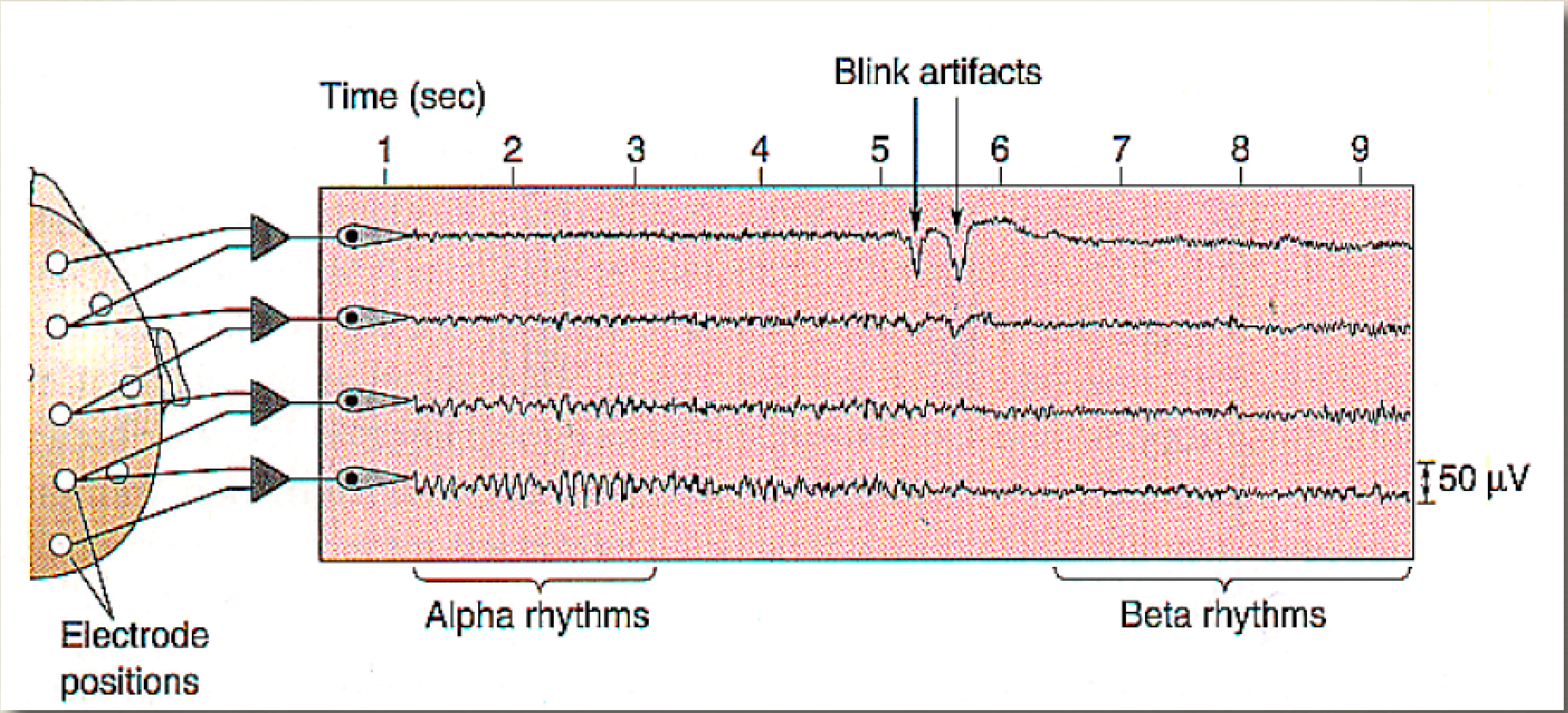
Drowsy

Light sleep

Deep sleep



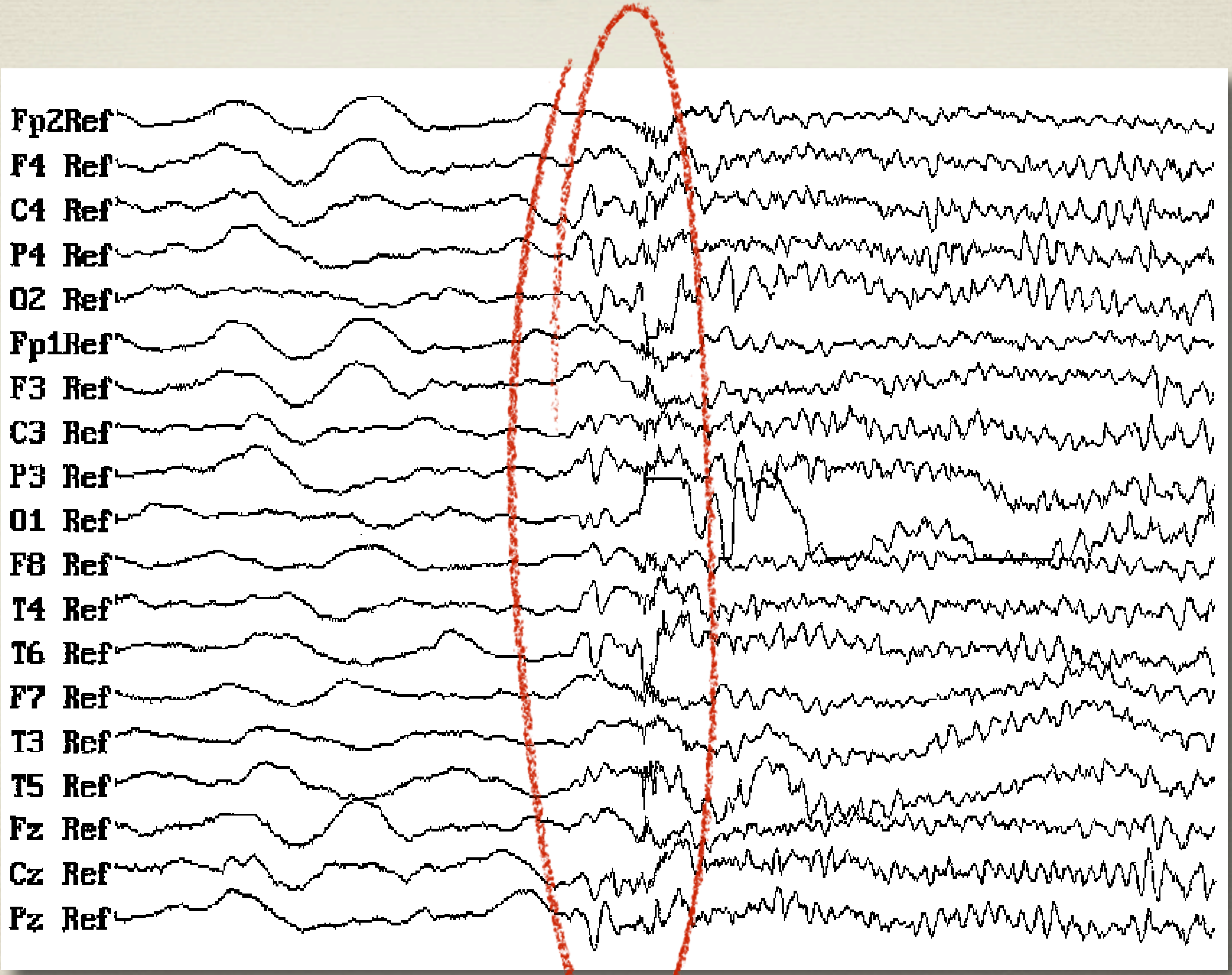
Alpha, Beta, and Blink Artifacts



The Use of EEG Today

* To investigate and diagnose
  * epilepsy (with related surgical procedures),
  * sleep disorders (many hour recordings),
  * dementia,
* To design a brain computer interface.
* Other applications as well, but less common.

Onset of Epileptic Seizure



 ording

The Soviets developed a jet fighter controlled by direct neural links, allowing an individual to pilot his jet by thought, without using his arm or leg muscles.

Brain Computer Interface (BCI) – Mental Prosthesis

* Goal: to give a severely handicapped person with normal mental capacity the ability to control a device that will provide communication with the environment.
* Two examples of devices are the speller and the wheelchair.

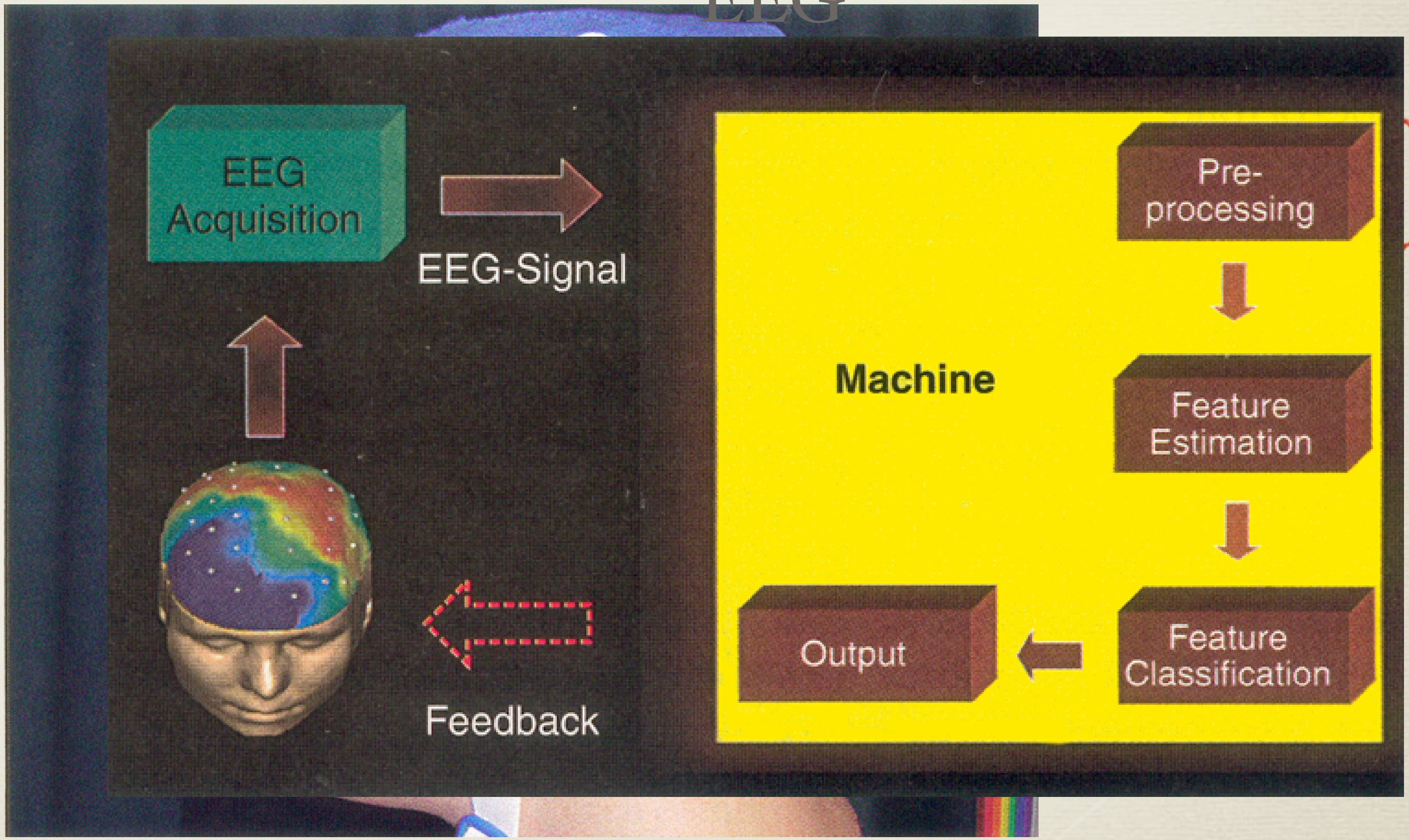
BCI Principle

- Detect changes in the EEG related to simple “mental” tasks which may be used to define an alphabet of actions.
- Changes may be expressed in terms of signal amplitude, phase, spectrum,...

Brain Computer Interface

EEG acquisition

EEG



BrainGate

EEG Modeling Aspects

* Stochastic versus deterministic
* Gaussian versus non-Gaussian
* Stationary versus nonstationary, with slow or abrupt changes, containing events
* Detail issues

Noise and Artifacts

* Any type of biomedical signal analysis must be preceded by noise rejection.
* The rejection is simplified when separate signals can be acquired not containing the desired signal (”reference signals”).
* EEG processing must deal with eye movements, cardiac activity, muscular activity, and other types of noise and artifacts.

EMG in the EEG

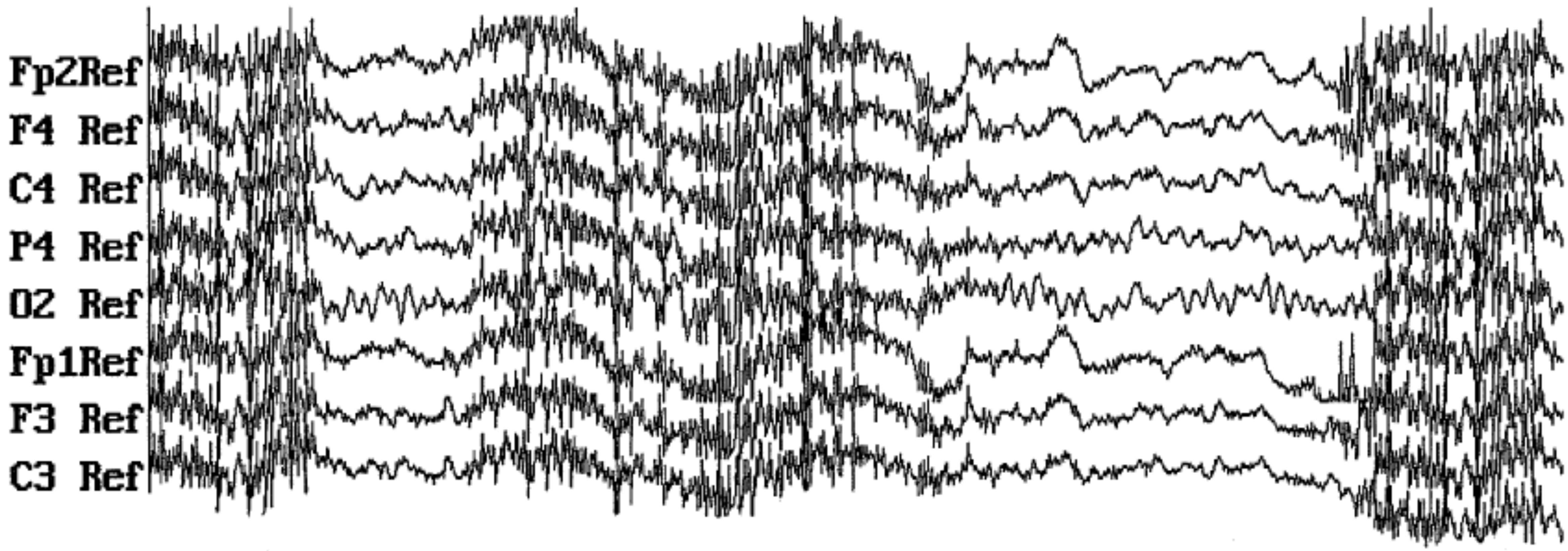
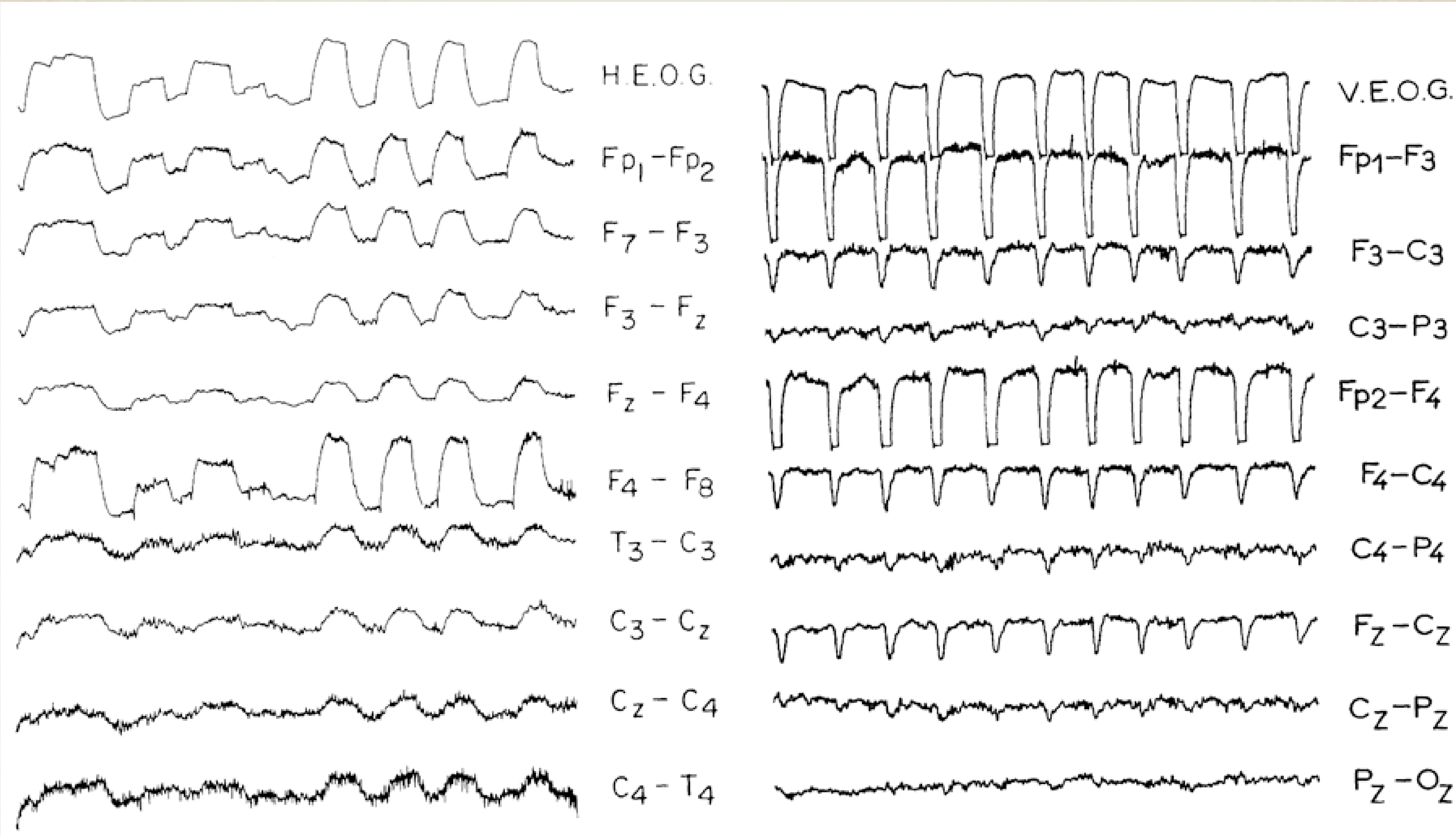


Figure 3.8: A 5-s, multichannel EEG recording contaminated with intermittent episodes of electromyographic artifacts. (Reprinted from Wong [66] with permission.)

Eye Movements and the EEG



"Optimal" Noise Rejection

* Basic idea:
  * estimate the noise of the observed signal by using a set of reference signals, and
  * subtract the resulting noise estimate from the observed signal.
* Linear model: weighting or filtering.
* Assumption: stationarity, ...

"Eye Movement" Electrodes

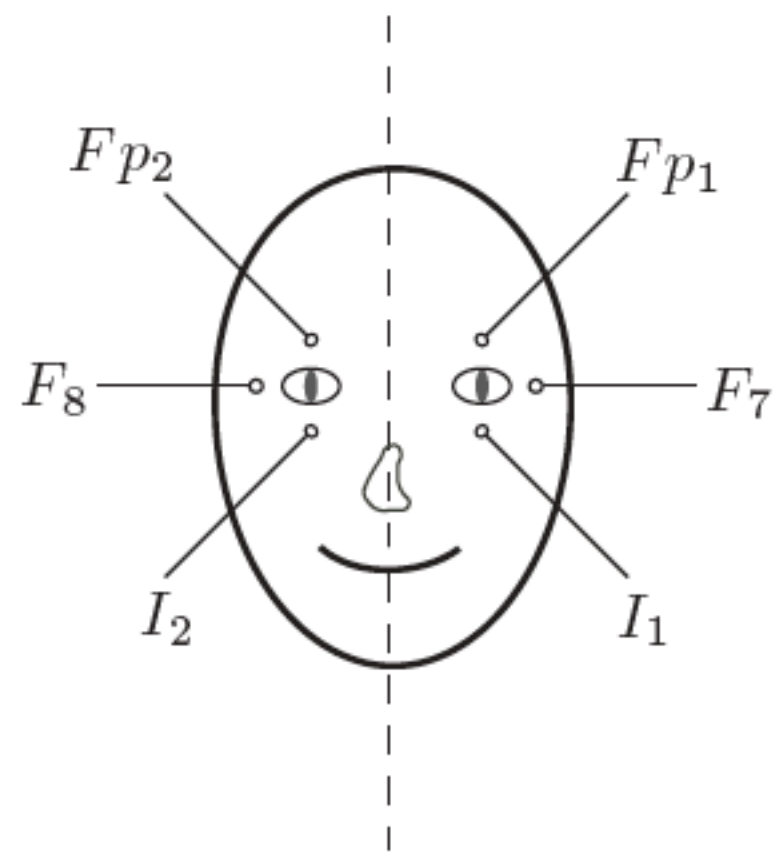
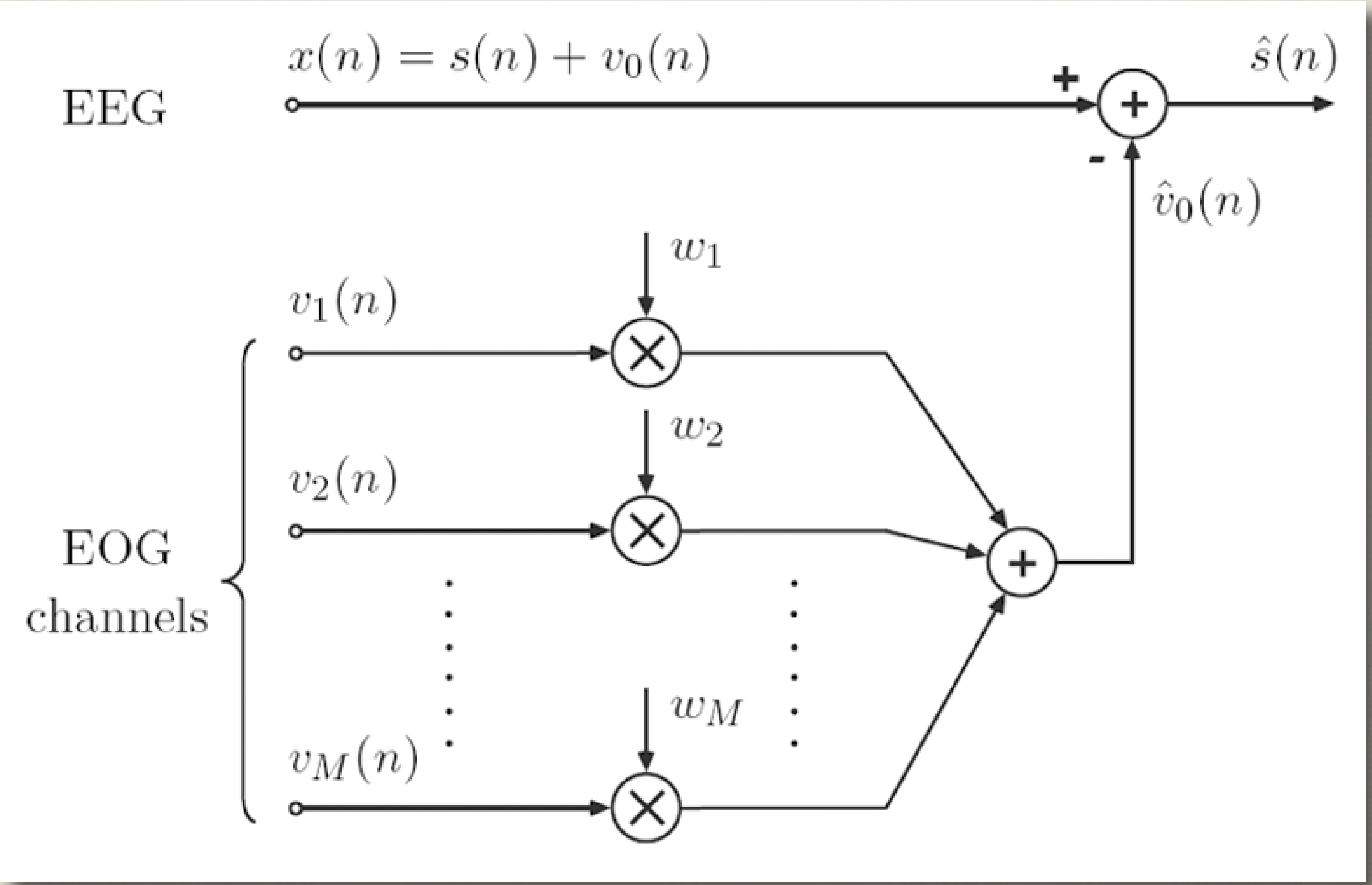


Figure 3.9: Electrode positions for the recording of EOG signals which reflect horizontal ($F_7 - F_8$) and vertical ($Fp_2 - I_2$ or $Fp_1 - I_1$) eye movement. Note that two other electrodes, I_1 and I_2, are used in addition to the electrodes of the 10/20 system shown in Figure 2.7.

Noise Rejection by Weighting



Linear Weighting

We assume that the EEG signal is composed of cerebral activity $s(n)$ which is additively disturbed by the EOG artifact $v_0(n)$,

$$x(n) = s(n) + v_0(n). \tag{3.30}$$

Another assumption in this approach is that the EOG reference signals $v_1(n), \ldots, v_M(n)$ are linearly transferred into the EEG signal. Hence, it seems reasonable to produce an artifact-cancelled signal $\hat{s}(n)$ by subtracting a linear combination of the reference signals from the EEG, using the weights $w_1, \ldots, w_M$,

$$\hat{s}(n) = x(n) - \sum_{i=1}^{M} w_i v_i(n) = s(n) + \left(v_0(n) - \mathbf{w}^T \mathbf{v}(n)\right), \tag{3.31}$$

Weights and The MSE

In the following, it is assumed that all signals are random in nature, with zero-mean, and that $s(n)$ is uncorrelated with the EOG signals $\mathbf{v}(n)$ at each time n,

$$E[s(n)v_i(n)] = 0, \quad i = 0, \ldots, M. \tag{3.34}$$

Mean Square Error (MSE):

$$\mathcal{E}_{\mathbf{w}} = E\left[\left(x(n) - \mathbf{w}^T\mathbf{v}(n)\right)^2\right]$$

or, equivalently

$$\mathcal{E}_{\mathbf{w}} = E\left[s^2(n)\right] + E\left[\left(v_0(n) - \mathbf{w}^T\mathbf{v}(n)\right)^2\right]$$

MSE Minimization

Differentiation of $\mathcal{E}_{\mathbf{w}}$ in (3.35) with respect to the coefficient vector $\mathbf{w}$ yields

$$\begin{aligned}\nabla_{\mathbf{w}}\mathcal{E}_{\mathbf{w}} &= \nabla_{\mathbf{w}}\left(E\left[x^2(n)\right] + \mathbf{w}^T\mathbf{R}_v(n)\mathbf{w} - 2\mathbf{w}^T\mathbf{r}_{xv}(n)\right)\\ &= 2\mathbf{R}_v(n)\mathbf{w} - 2\mathbf{r}_{xv}(n).\end{aligned} \tag{3.37}$$

The correlation matrix $\mathbf{R}_v(n)$ of the reference signals describes the *spatial correlation* between the different channels at each time n and is defined by

$$\begin{aligned}\mathbf{R}_v(n) &= E\left[\mathbf{v}(n)\mathbf{v}^T(n)\right]\\ &= \begin{bmatrix} r_{v_1v_1}(n) & r_{v_1v_2}(n) & \cdots & r_{v_1v_M}(n)\\ r_{v_2v_1}(n) & r_{v_2v_2}(n) & \cdots & r_{v_2v_M}(n)\\ \vdots & \vdots & & \vdots\\ r_{v_Mv_1}(n) & r_{v_Mv_2}(n) & \cdots & r_{v_Mv_M}(n)\end{bmatrix},\end{aligned} \tag{3.38}$$

Assumption of Stationarity

Although the correlation quantities $\mathbf{R}_v(n)$ and $\mathbf{r}_{xv}(n)$ change over time, we will for now assume that these quantities remain fixed over the observation interval of interest,

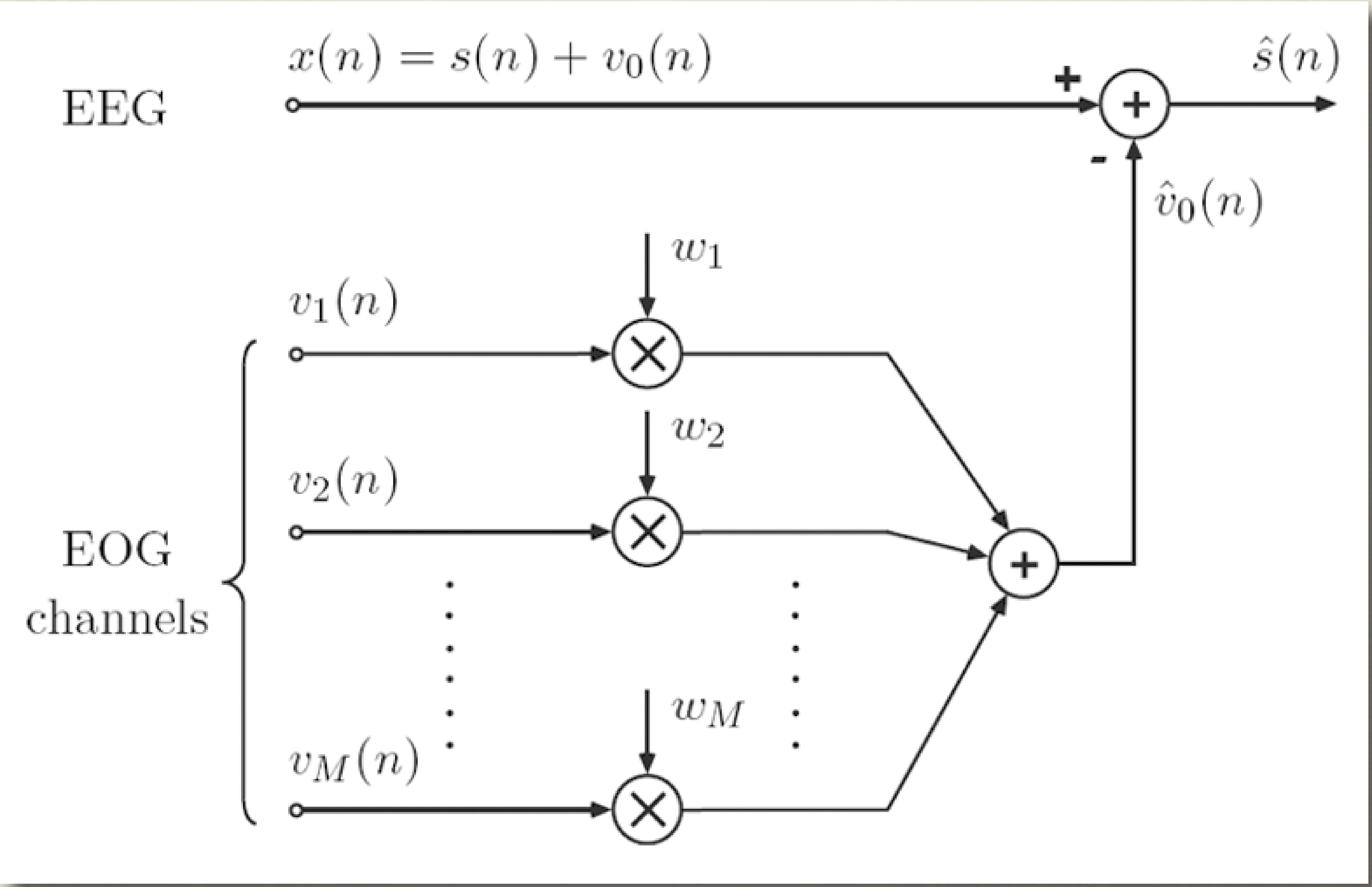
$$\mathbf{R}_v(n) \equiv \mathbf{R}_v, \tag{3.42}$$

$$\mathbf{r}_{xv}(n) \equiv \mathbf{r}_{xv}, \tag{3.43}$$

Setting the gradient $\nabla_{\mathbf{w}}\mathcal{E}_{\mathbf{w}}$ in (3.37) equal to zero, we obtain the following system of linear equations,

$$\mathbf{R}_v \mathbf{w}^{\mathrm{o}} = \mathbf{r}_{xv}, \tag{3.44}$$

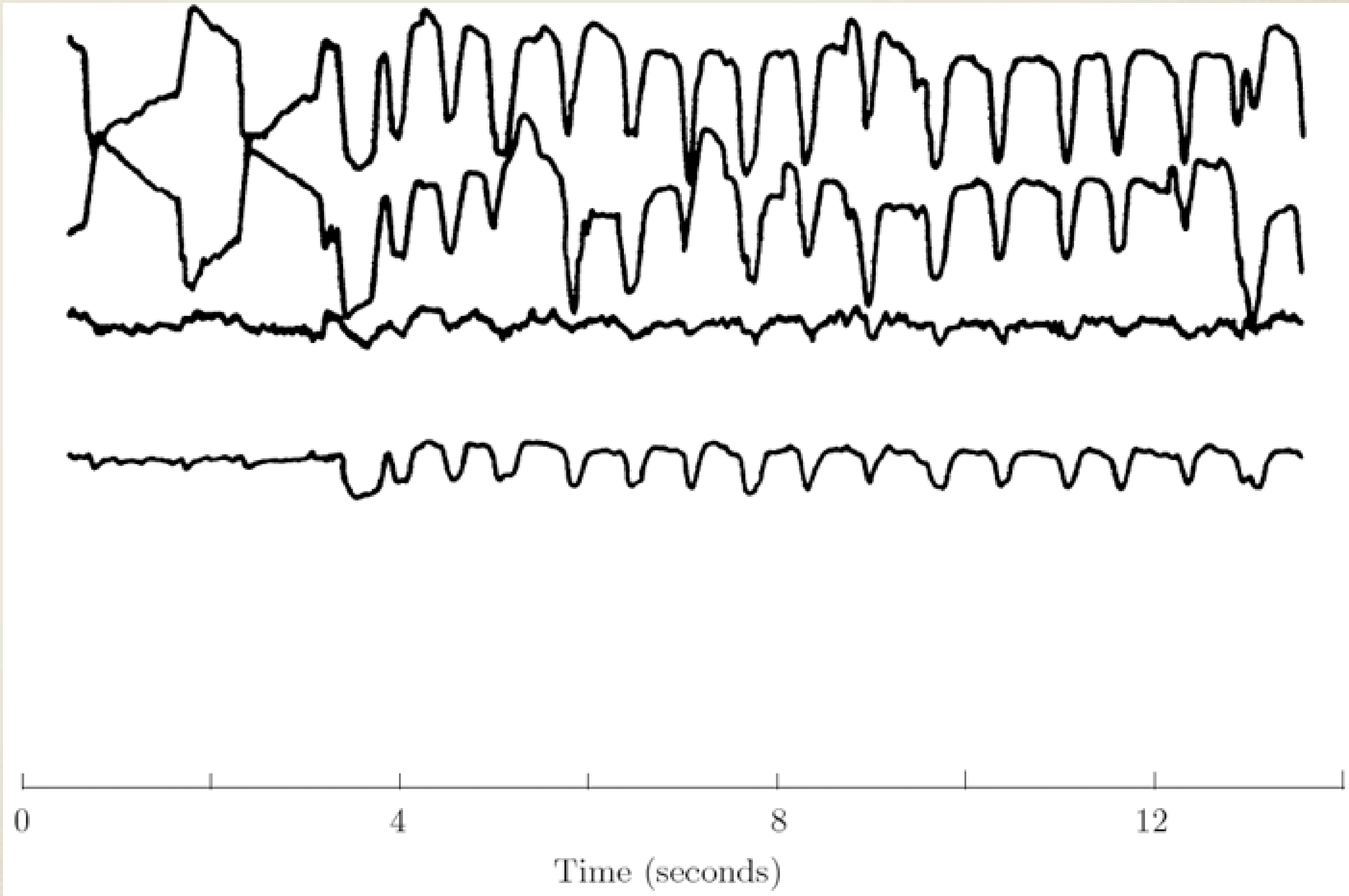
Noise Rejection by Weighting



Correction of ElectroOculoGram

EOG
left/right eye

EEG
before
correction

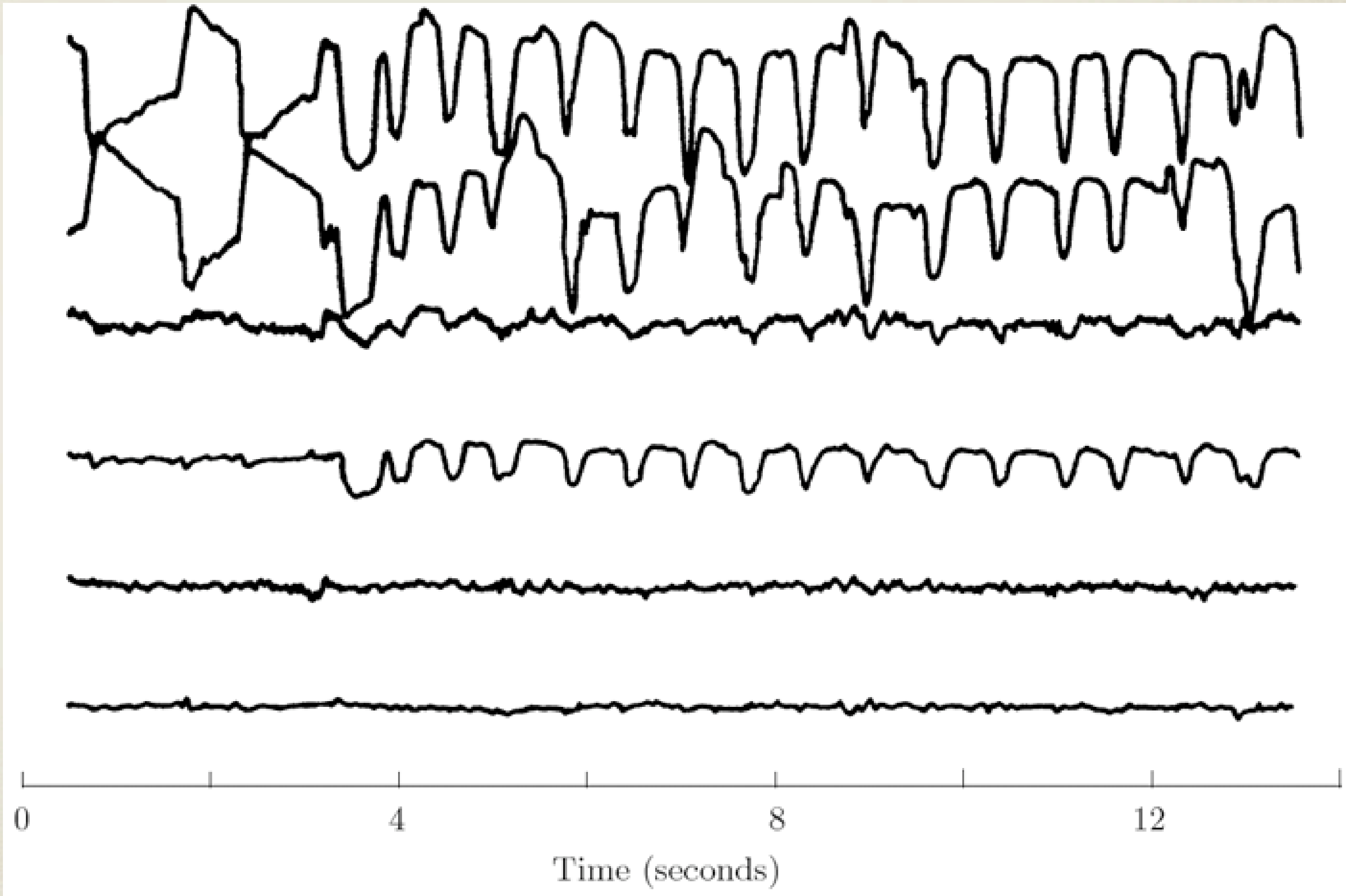


Correction of ElectroOculoGram

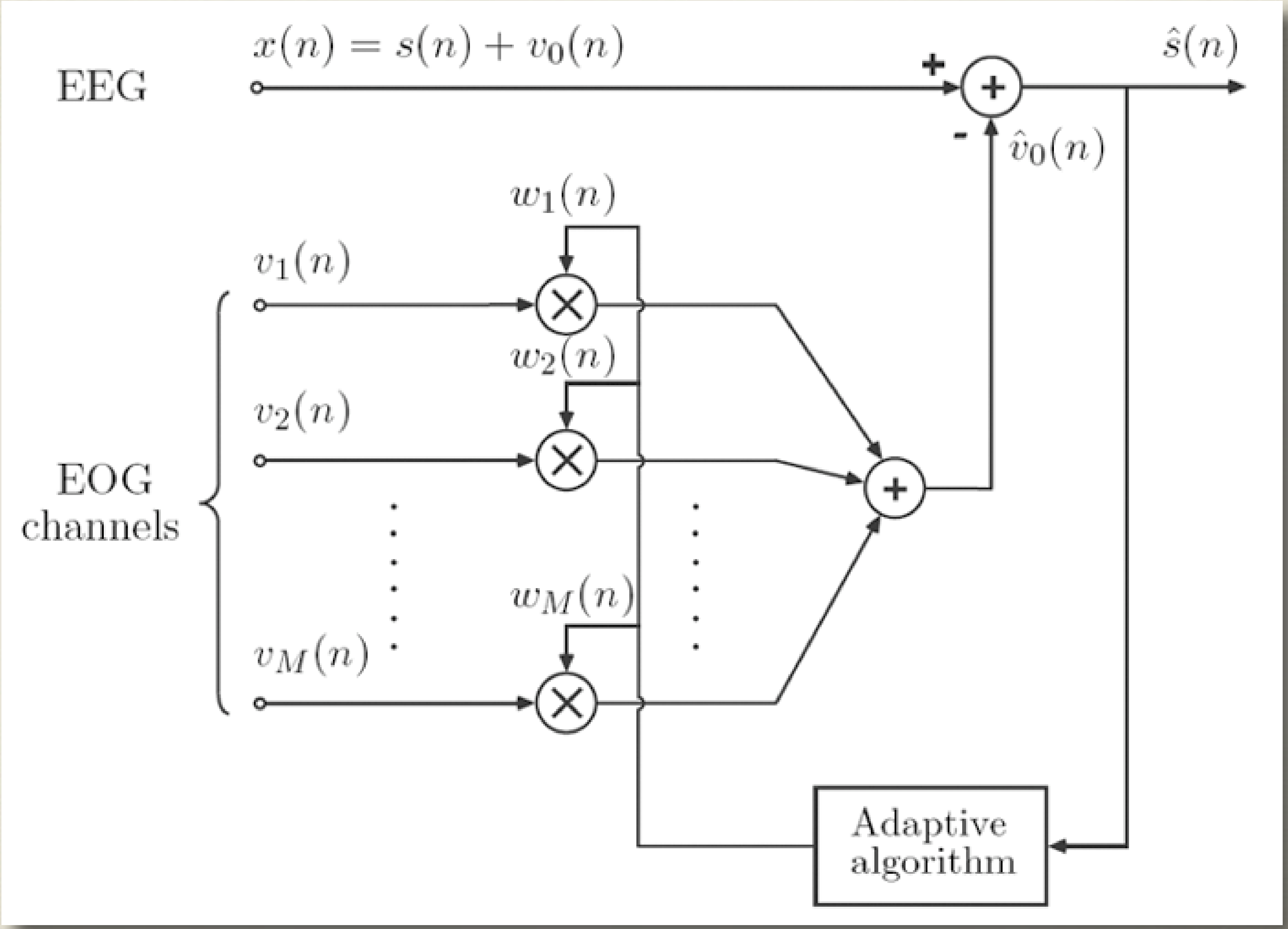
EOG
left/right eye

EEG
before
correction

EEG
after
correction

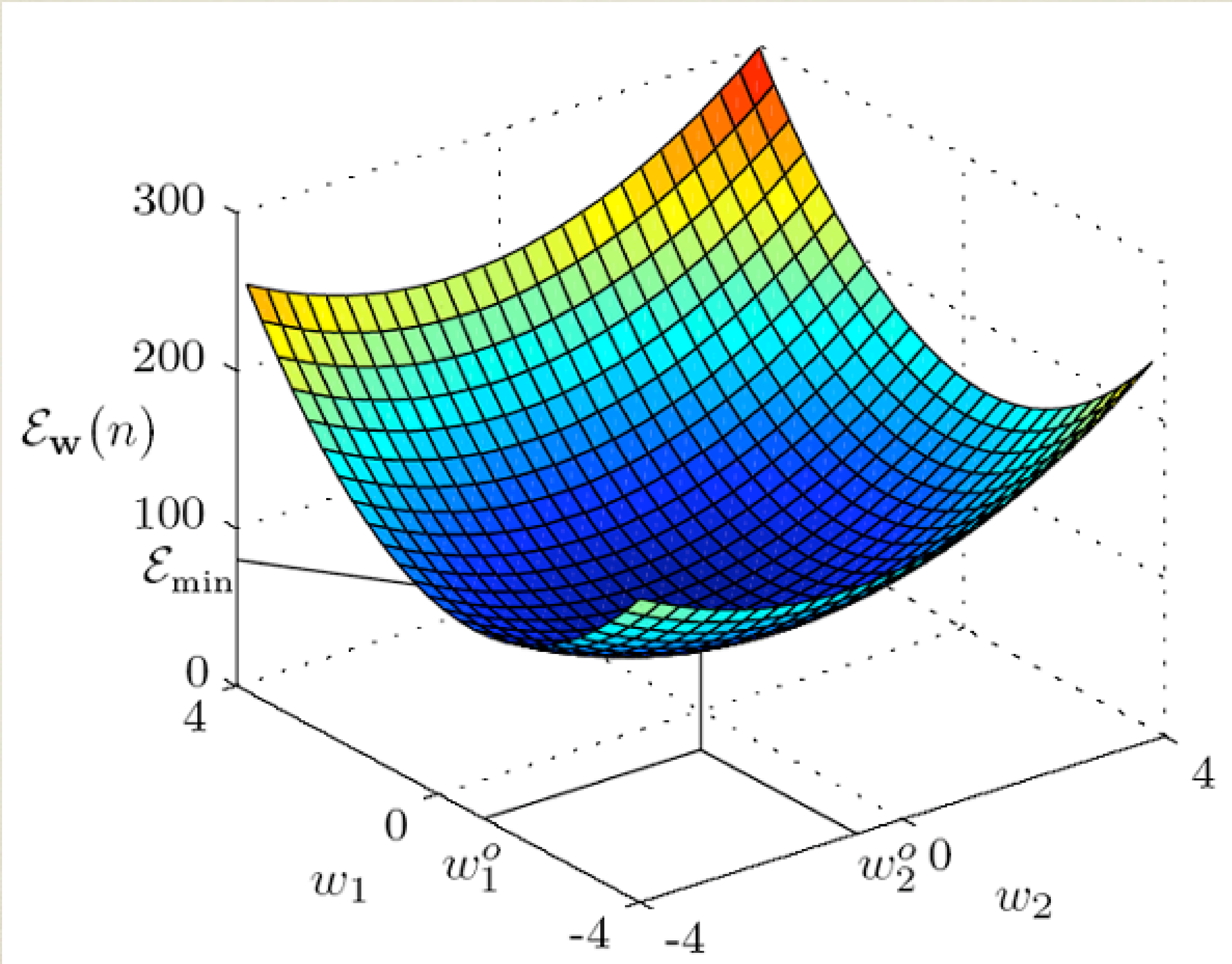


Adaptive Noise Rejection

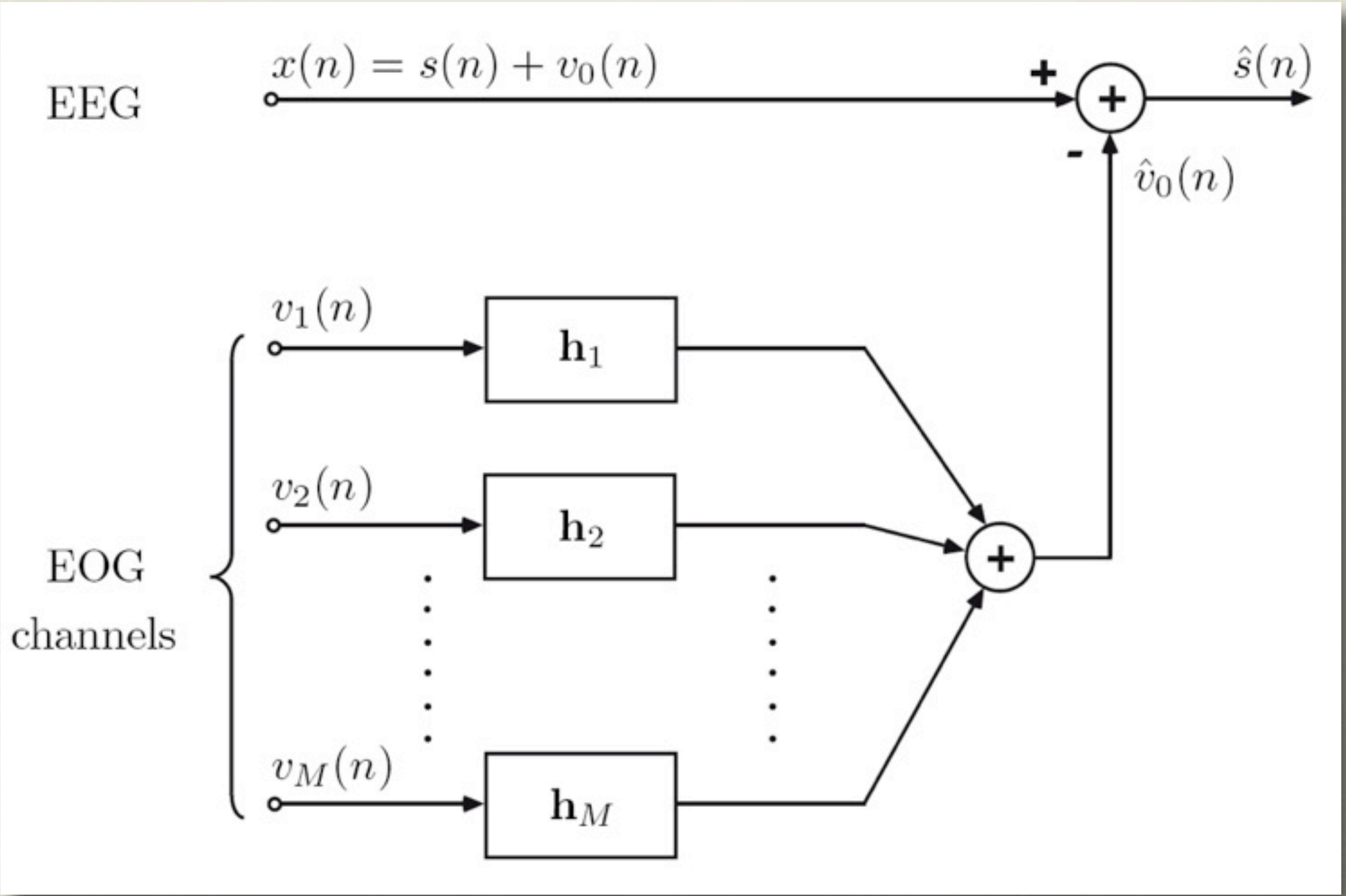


$$\mathbf{w}(n+1) = \mathbf{w}(n) - \frac{1}{2}\mu \nabla_{\mathbf{w}} \mathcal{E}_{\mathbf{w}}(n)$$

MSE Criterion Minimization



Noise Rejection – Filtered Reference Signals



Spectral Analysis of the EEG

- Spectral analysis based on the stationarity assumption:
  - non-parametric, Fourier-based analysis, or
  - parametric analysis based on AR modeling.
- Time-frequency analysis is suitable for non-stationary signals.

Spectral Analysis of EEG

The analysis may be based on...

- the Fourier transform (the periodogram)

$$S_x(e^{j\omega}) = \sum_{k=-\infty}^{\infty} r_x(k) e^{-j\omega k}$$

- or on a linear, stochastic model such as the autoregressive (AR) model:

$$x(n) = -a_1 x(n-1) - \ldots - a_p x(n-p) + v(n)$$

Fourier-Based Spectral Analysis

* How to estimate the power spectrum from the observed signal?
* What are the properties of the spectral estimator?
* Variations on the basic periodogram method to improve its performance.

The Periodogram

$$S_x(e^{\jmath\omega}) = \sum_{k=-\infty}^{\infty} r_x(k) e^{-\jmath\omega k},$$

$$\hat{r}_x(k) = \frac{1}{N} \sum_{n=0}^{N-1-k} x(n+k)x(n), \quad k = 0, \ldots, N-1,$$

$$\hat{S}_x(e^{\jmath\omega}) = \sum_{k=-N+1}^{N-1} \hat{r}_x(k) e^{-\jmath\omega k},$$

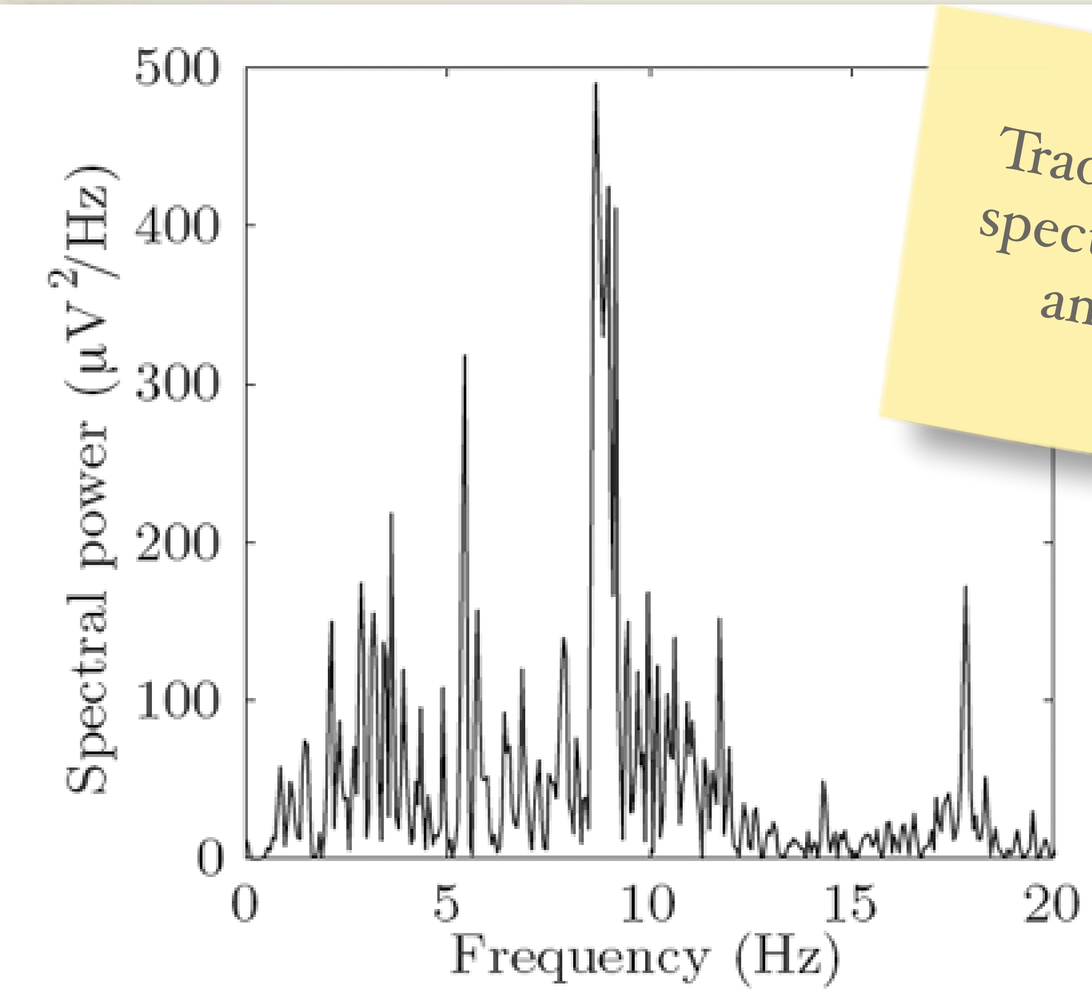
$$\hat{S}_x(e^{\jmath\omega}) = \frac{1}{N} |X(e^{\jmath\omega})|^2 = \frac{1}{N} \left| \sum_{n=0}^{N-1} x(n) e^{-\jmath\omega n} \right|^2$$

Properties of the Periodogram

* Spectral leakage and...
* large variance...
* ...are combatted by the use of windowing and segmentation & averaging – Welch's method.

Periodogram & Segmentation

5-s EEG with alpha rhythm



Trade-off between spectral resolution and variance

no segmentation

segment length: 256 / overlap: 128

Spectral Parameters

* Power in different spectral bands.
* Location, amplitude, and width of one or several spectral peaks.
* Spectral moments and related measures such as the Hjorth descriptors.

Spectral Parameters

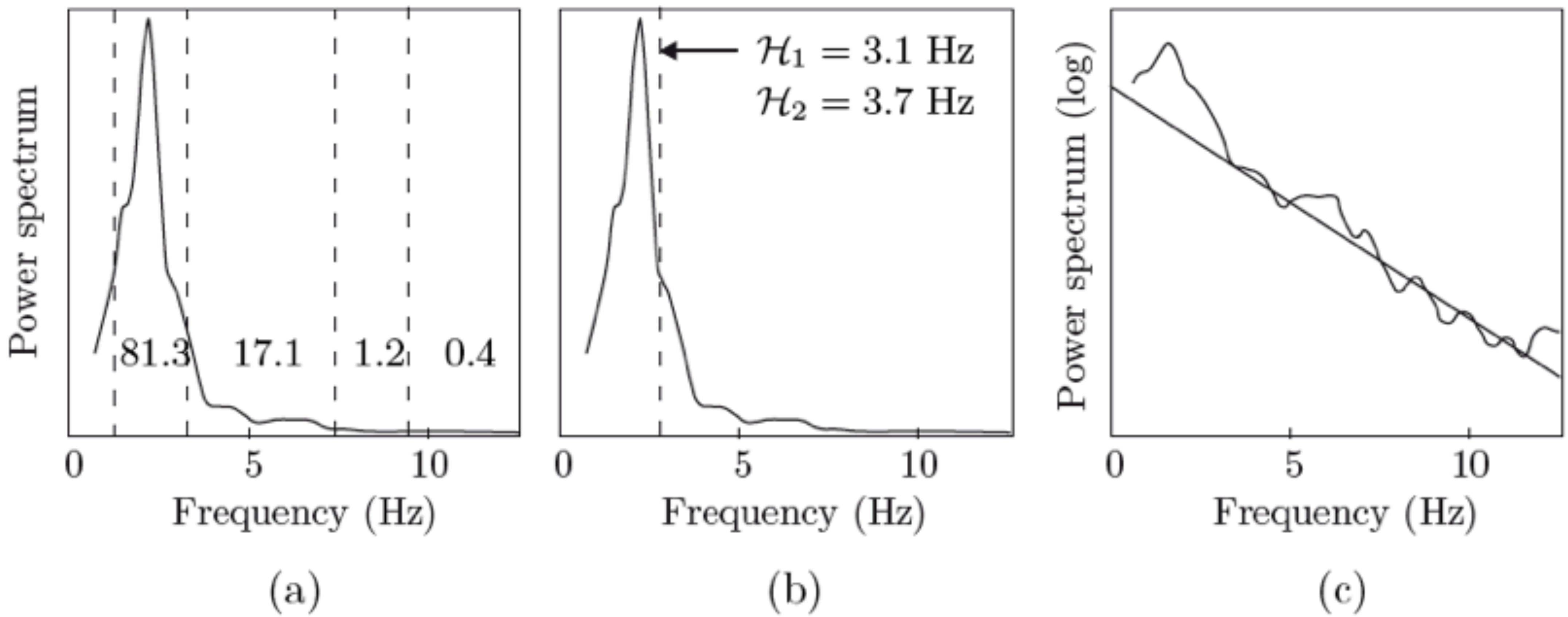
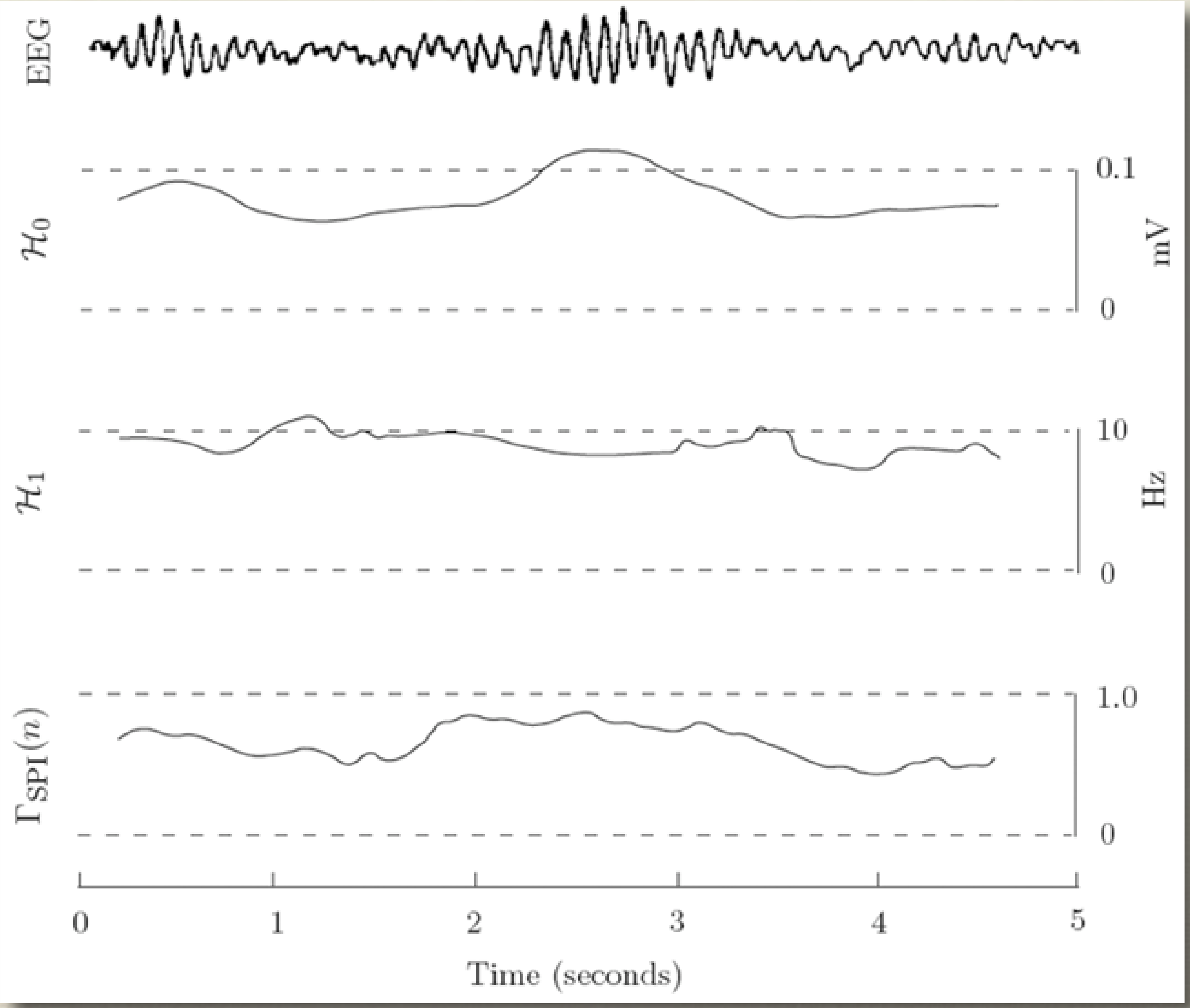


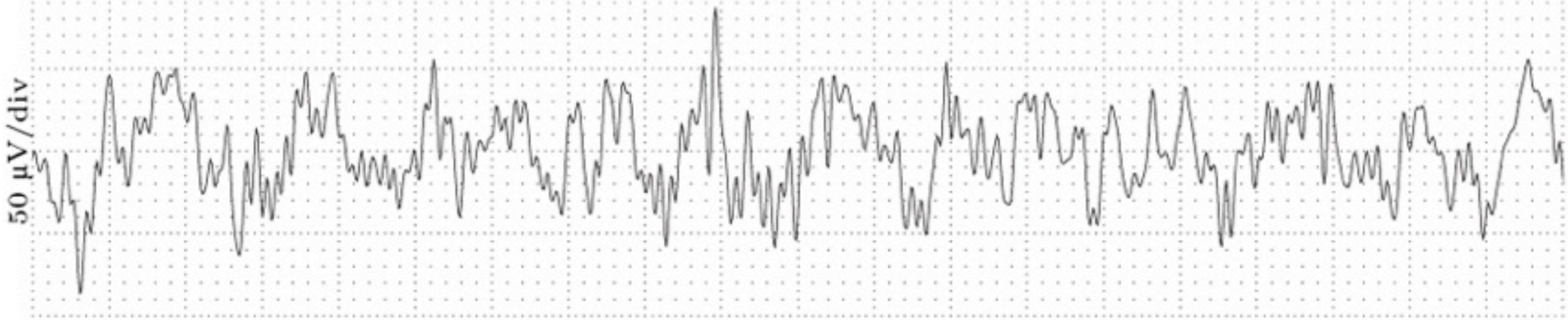
Figure 3.17: The power spectrum of an EEG and related parameters: (a) relative power (percentages) in four frequency bands reflecting delta, theta, and alpha activity (split into two bands); (b) the Hjorth parameters mobility $\mathcal{H}_1$ and complexity $\mathcal{H}_2$; and (c) the spectral slope estimated from the logarithmic power spectrum. The EEG was recorded from a child with a brainstem tumor and is dominated by slow rhythmic activity as reflected by the value of $\mathcal{H}_1$ of 3.1 Hz. (Adapted from Matthis et al. [121]).

Trending of Spectral Parameters



EEG and AR Modeling

EEG with alpha rhythm



Signal produced by an autoregressive (AR) model

(parameters estimated from the upper signal)

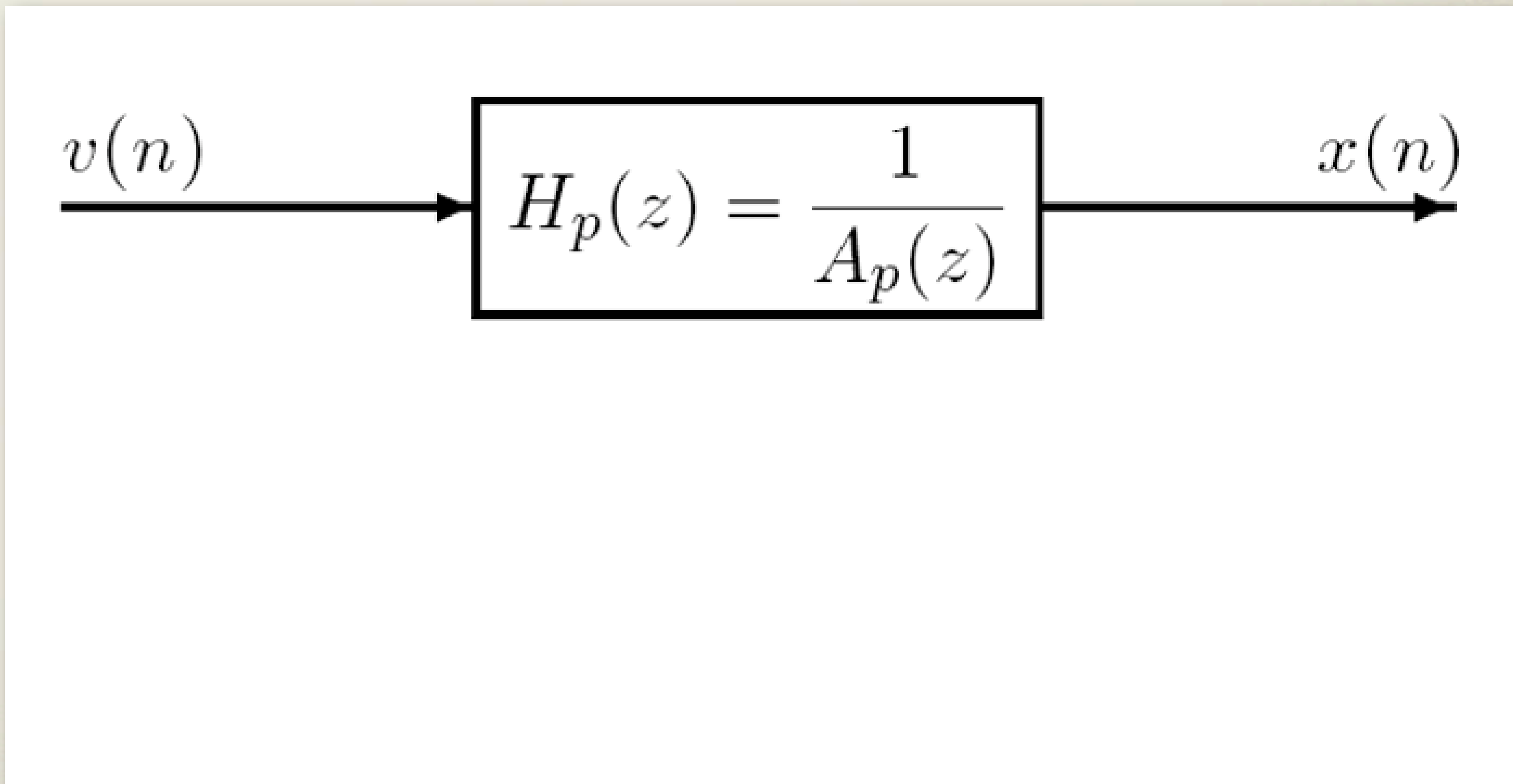
Autoregressive (AR) Modeling

- Suitable for EEG rhythms which have spectra with a ”peaky” shape.
- Several parameter estimation methods exist which are based on the linear prediction idea.
- Minimize the prediction error with respect to the AR parameters in the mean square error (MSE) sense.

AR Modeling and Linear Prediction

Model

Predictor



AR Modeling and Linear Prediction, cont'

AR model:

$$x(n) = -a_1 x(n-1) - \cdots - a_p x(n-p) + v(n)$$

FIR predictor:

$$\hat{x}_p(n) = -a_1 x(n-1) - \cdots - a_p x(n-p).$$

Prediction error:

$$\begin{aligned} e_p(n) &= x(n) - \hat{x}_p(n) \\ &= x(n) + \sum_{k=1}^{p} a_k x(n-k), \end{aligned}$$

MSE criterion

$$\sigma_e^2 = E\left[e_p^2(n)\right],$$

AR Parameter Estimation

* Estimation methods include:
  * the autocorrelation/covariance method
  * the forward/backward method
  * Burg's method
  * (others not described in the textbook)
* Critical issue: model order determination

The Normal Equation

The AR coefficients are obtained from the following matrix equation:

$$\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix} = \begin{bmatrix} r_x(0) & r_x(1) & \cdots & r_x(p-1) \\ r_x(1) & r_x(0) & \cdots & r_x(p-2) \\ \vdots & \vdots & \ddots & \vdots \\ r_x(p-1) & r_x(p-2) & \cdots & r_x(0) \end{bmatrix}^{-1} \begin{bmatrix} -r_x(1) \\ -r_x(2) \\ \vdots \\ -r_x(p) \end{bmatrix}, \qquad (3.127)$$

and the white noise variance from:

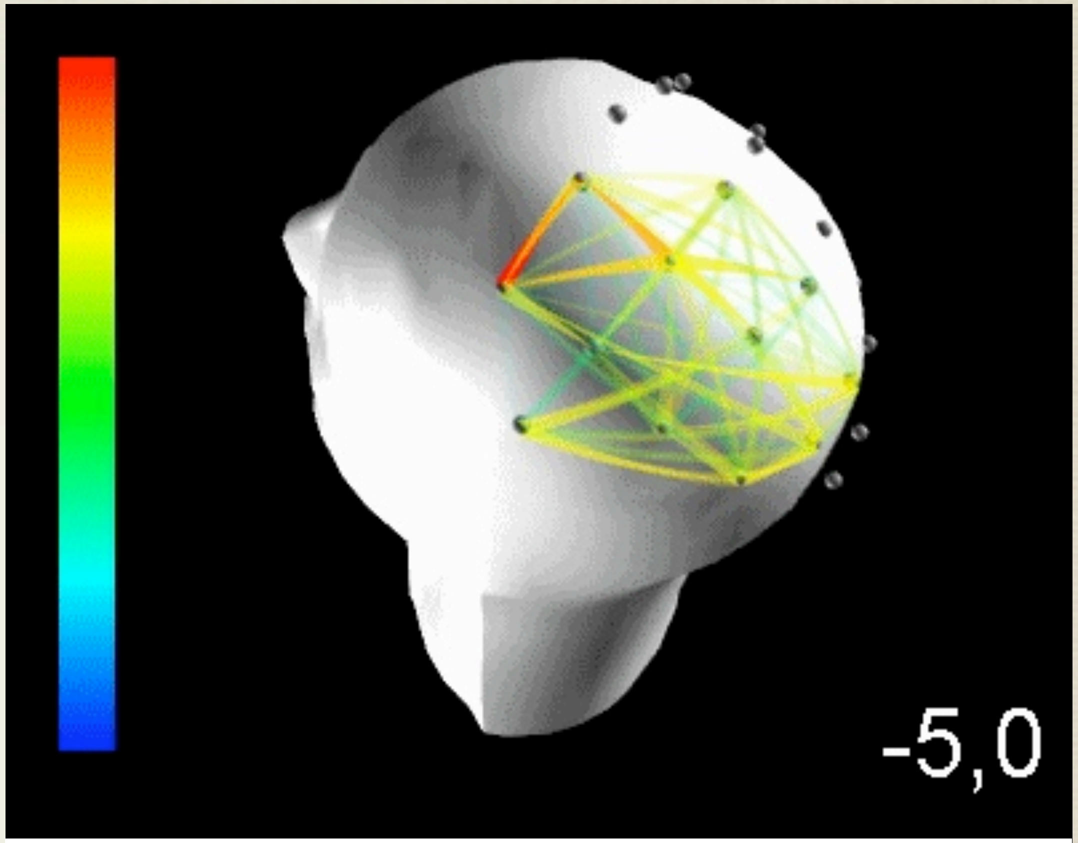
$$\sigma_e^2 = r_x(0) + \sum_{i=1}^{p} a_i r_x(i). \qquad (3.128)$$

Multivariate AR models

- Multivariate AR (MVAR) modeling is very common within neuroscience to evaluate causality for brain signals, e.g., during epilepsy (not covered in this course).
- Based on the Fourier transform of the estimated MVAR model, several measures of causality can be defined in the frequency domain.
- The next slide illustrates the information flow in cortex during finger movement, as reflected by the EEG.

EEG Activity Propagation After Finger Movement

Alpha rhythm

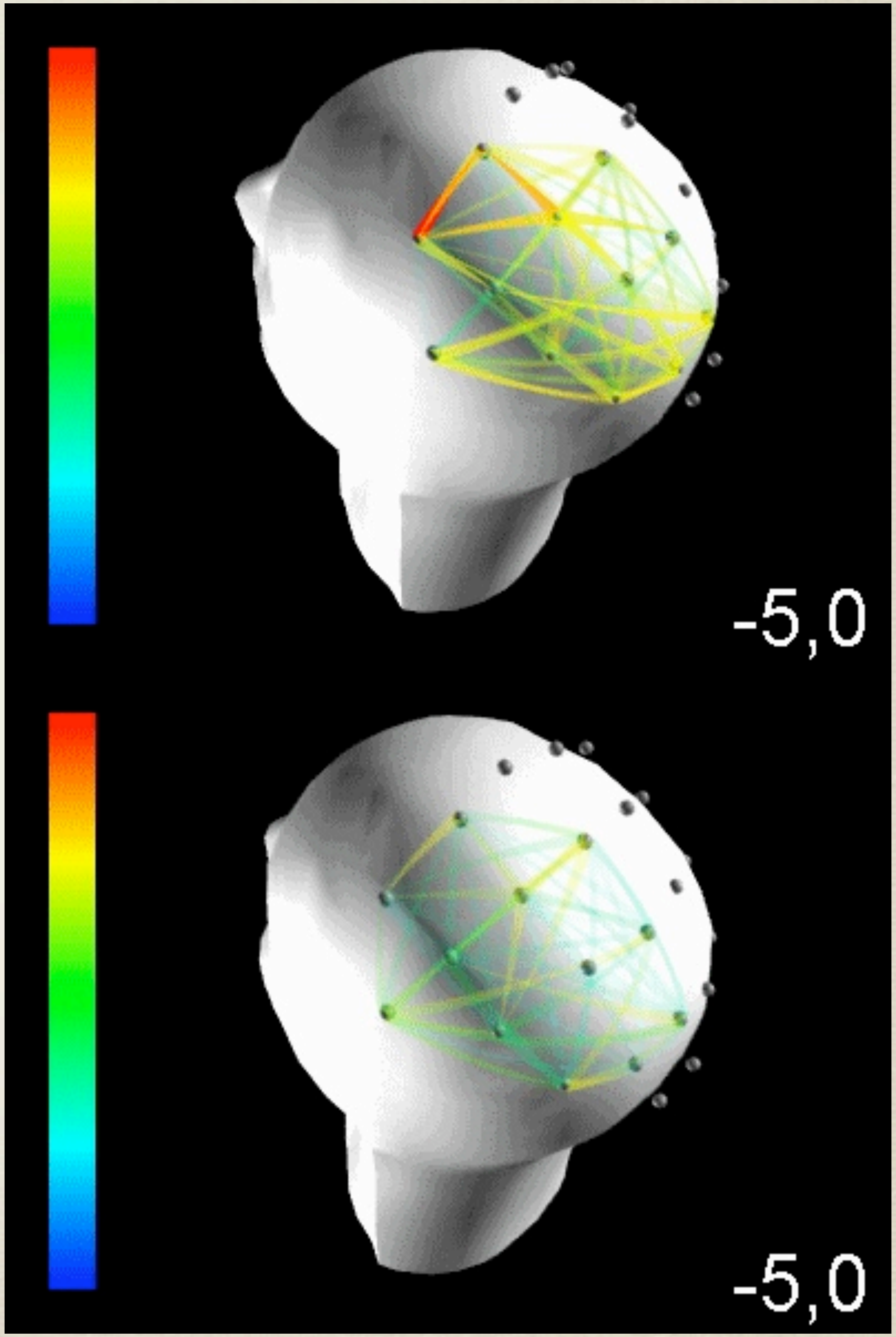


Alpha rhythm starting 5 seconds before the finger movement, and lasting 3 seconds after.

EEG Activity Propagation After Finger Movement

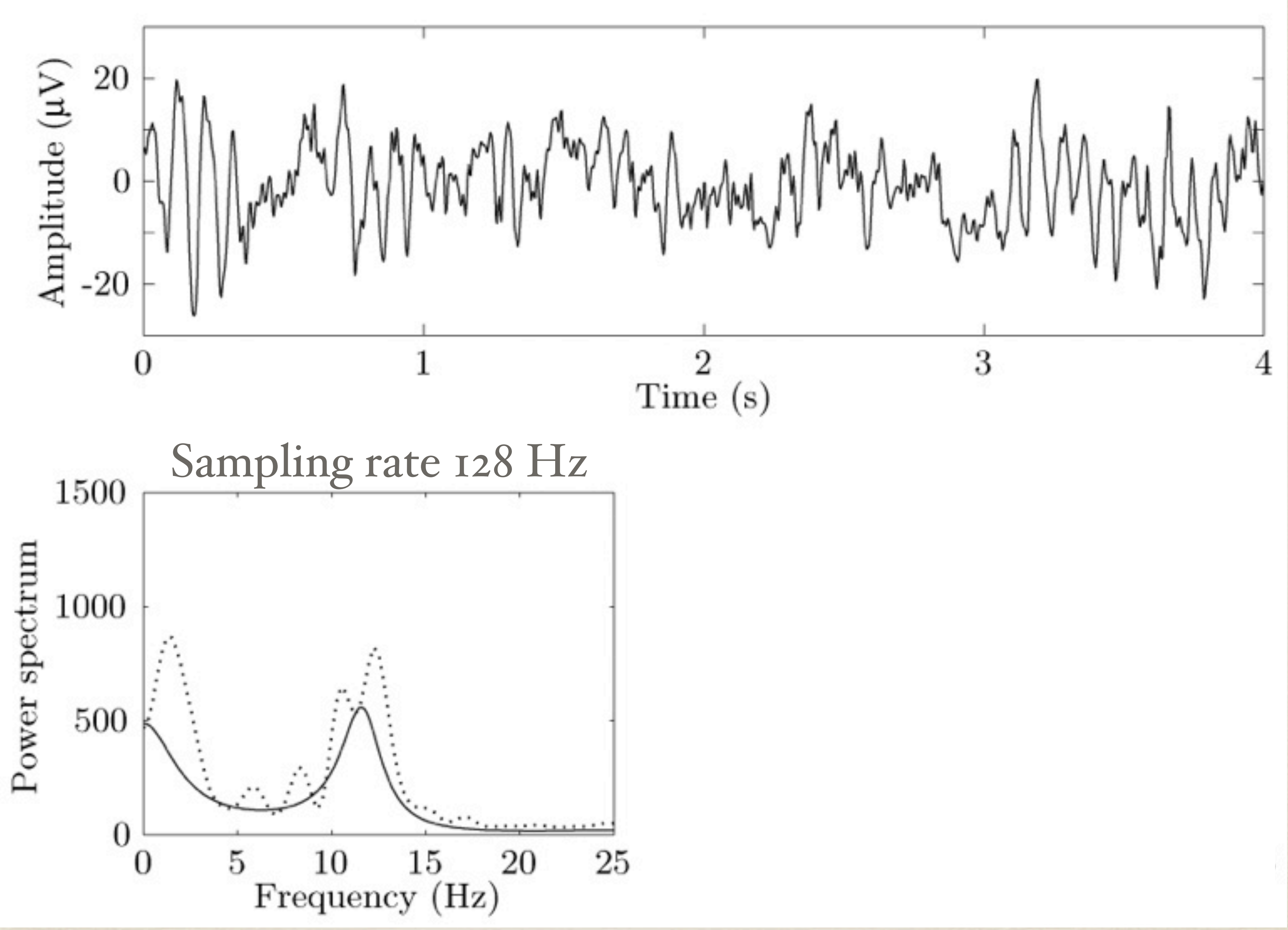
Alpha rhythm

Beta rhythm



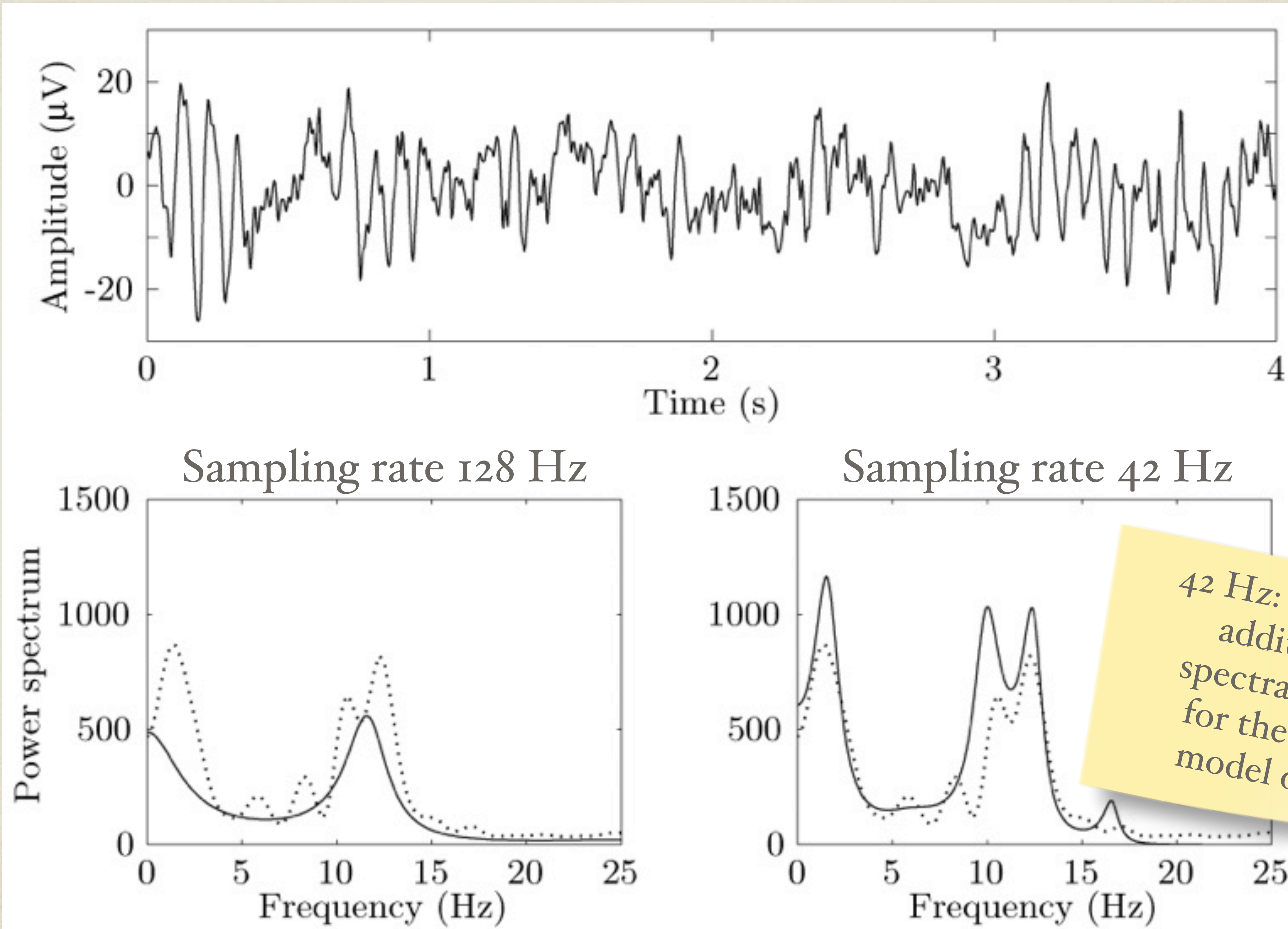
Visit eeg.pl for more info

AR Modeling and Sampling Rate



dotted line – spectrum obtained by Welch's method

AR Modeling and Sampling Rate



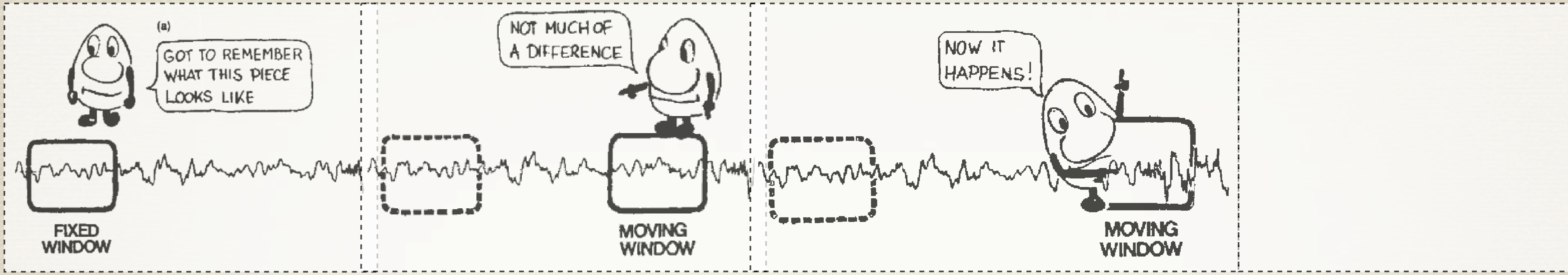
42 Hz: Note the additional spectral detail for the same model order

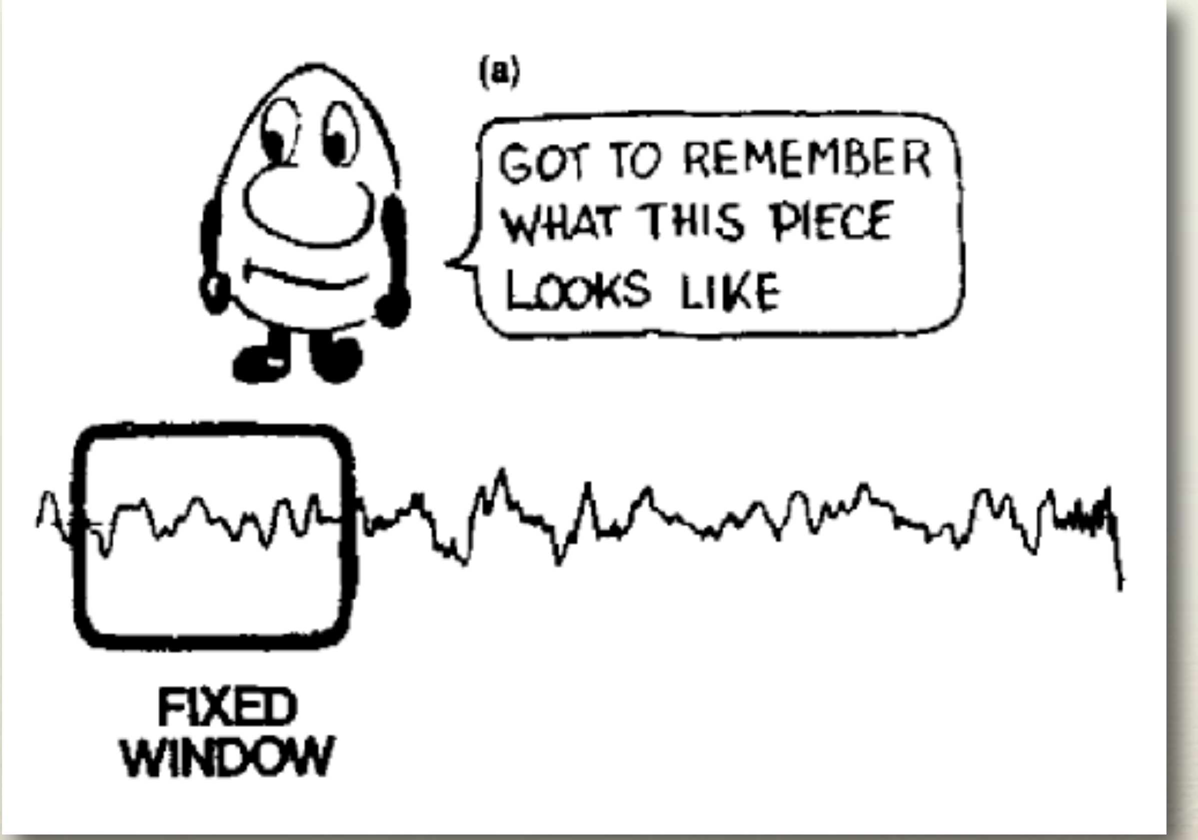
dotted line – spectrum obtained by Welch's method

Adaptive EEG Segmentation

* Assumption: piecewise stationary signal.
* Study: spectral changes.
* Required: definition of a measure indicating a change of the power spectrum.
* Result: segments which are relevant for the clinician (or, at least, for an automated classification program...).

Segmentation–Sliding Window





Criterion for EEG Segmentation

A straightforward approach is to make use of a **spectral MSE criterion**

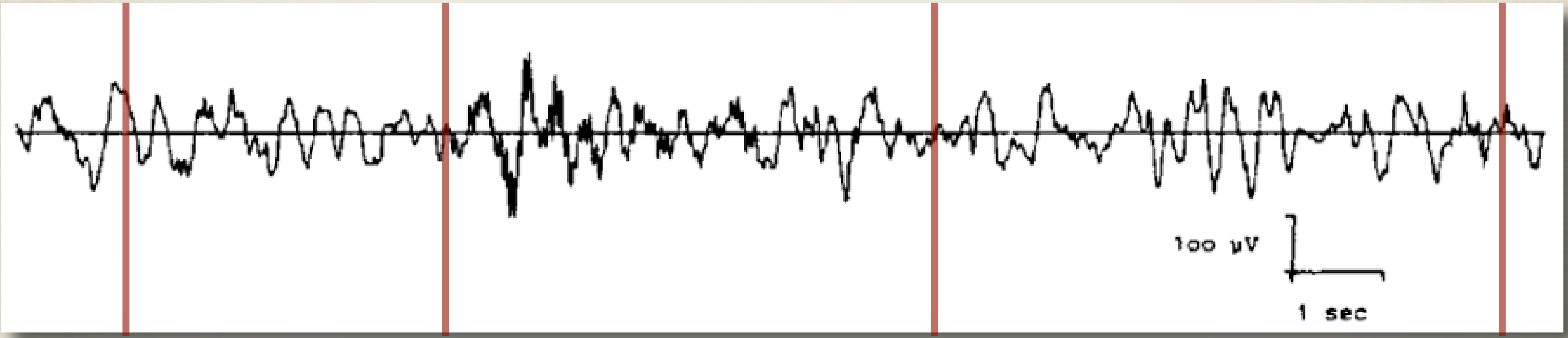
$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\left(S_x(e^{j\omega},n)-S_x(e^{j\omega},0)\right)^2 d\omega.$$

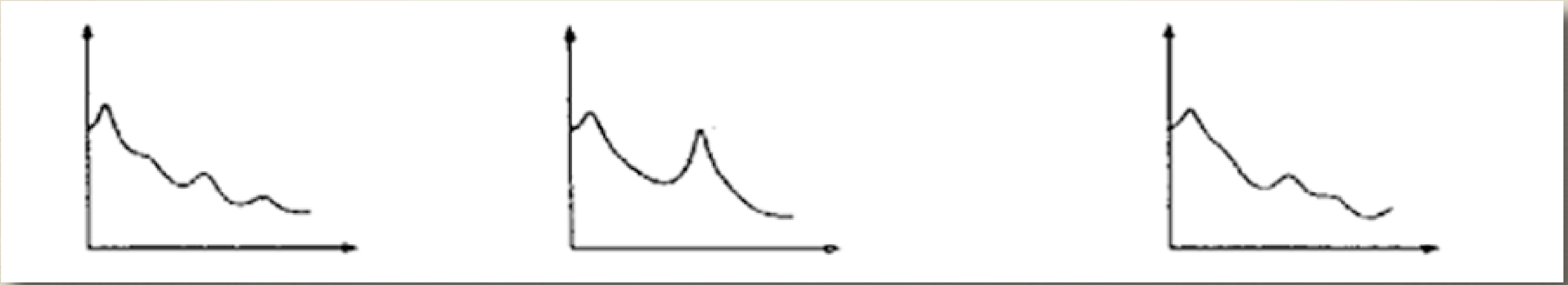
where

$$S_x(e^{j\omega},n)=\sum_{k=-N+1}^{N-1} r_x(k,n)e^{-j\omega k}$$

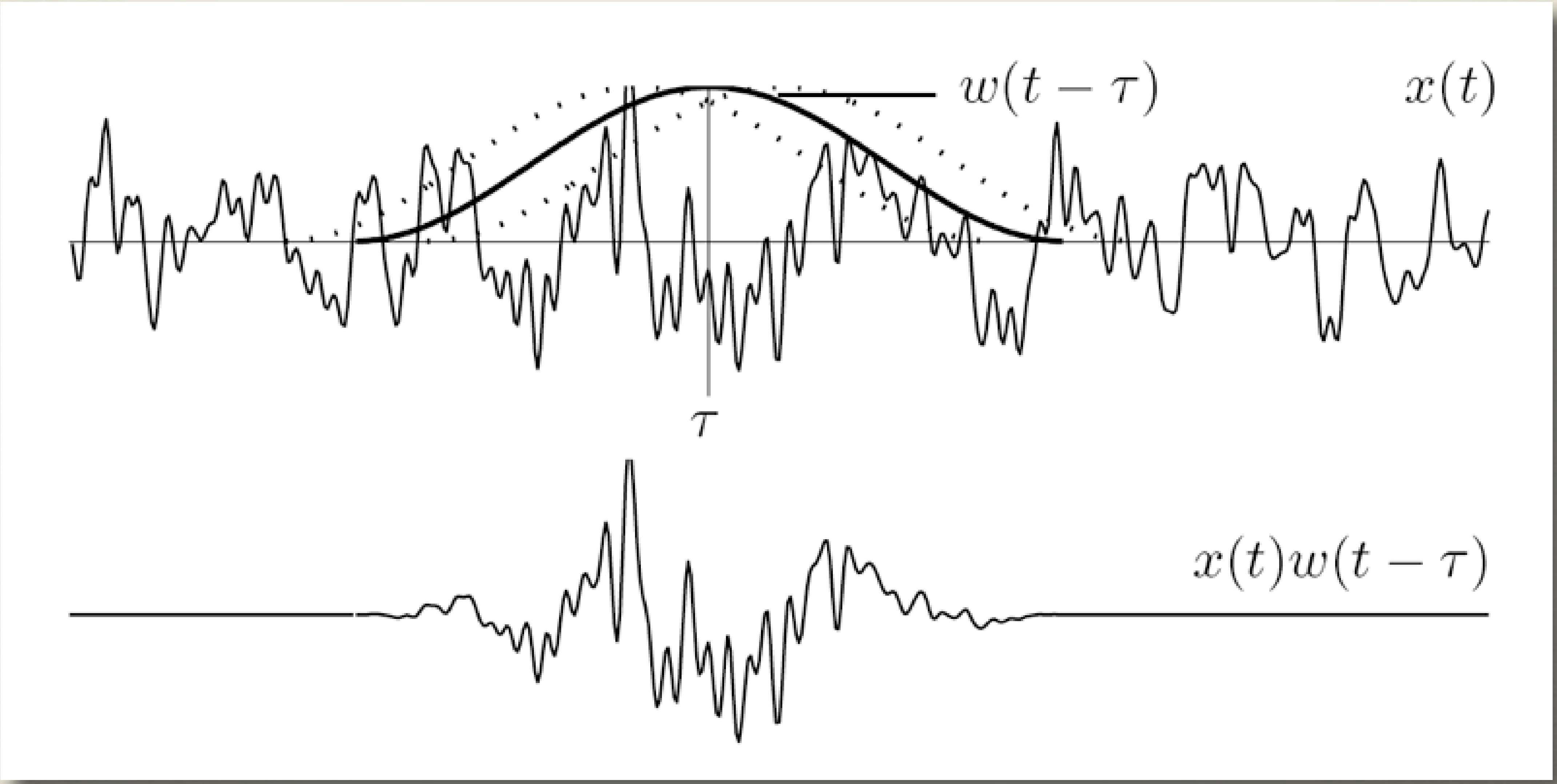
Disadvantages?

Spectra of a Segmented Signal





Spectral Analysis of Nonstationary Signals



Short-Time Fourier Transform (STFT)

The Fourier transform in continuous time

$$X(\Omega) = \int_{-\infty}^{\infty} x(\tau) e^{-j\Omega\tau} d\tau$$

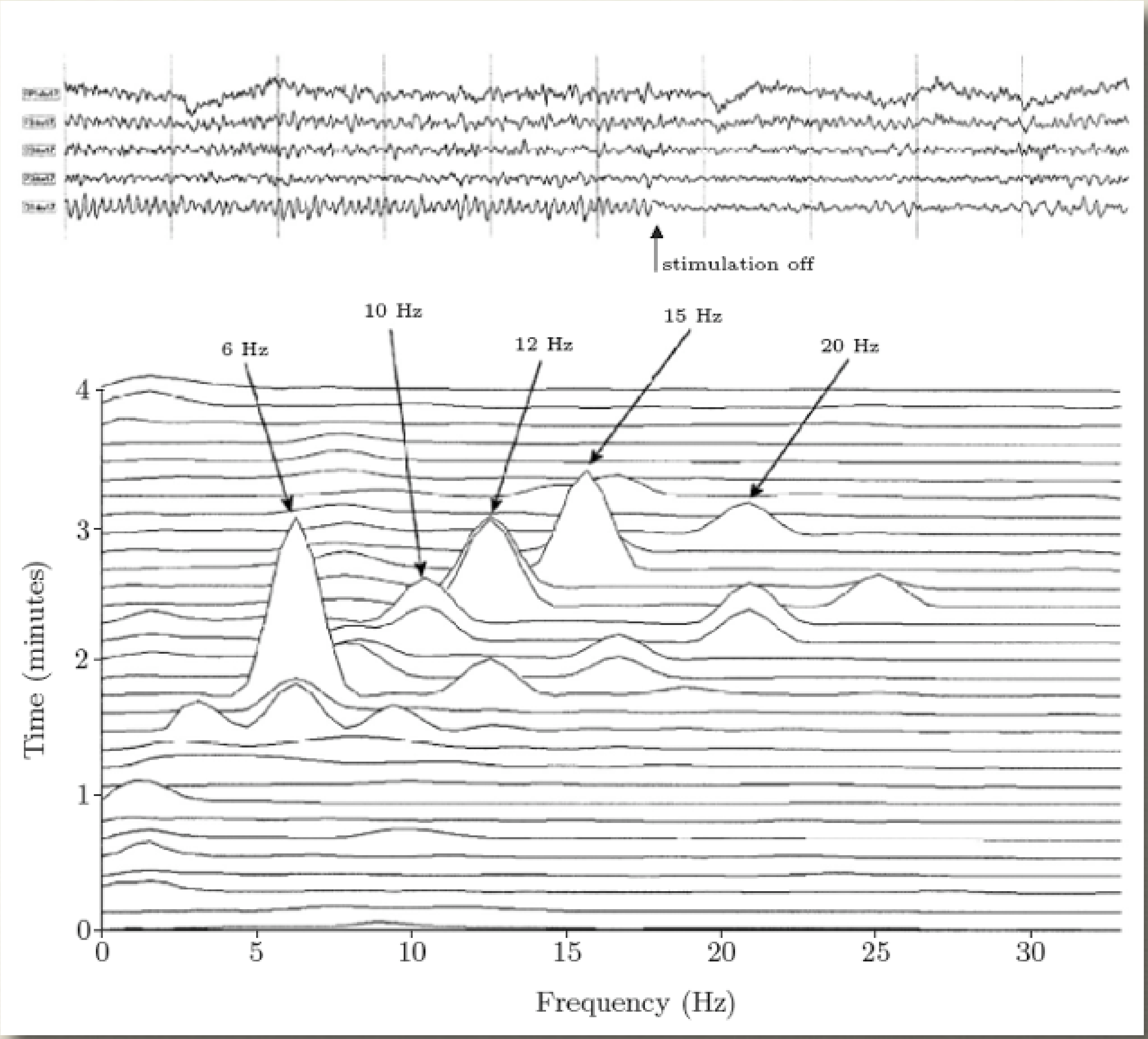
is extended such that a 2-dimensional function is obtained which describes frequency content at different times:

$$X(t, \Omega) = \int_{-\infty}^{\infty} x(\tau) w(\tau - t) e^{-j\Omega\tau} d\tau$$

Photic Stimulation at Different Rates

4-channel EEG

STFT

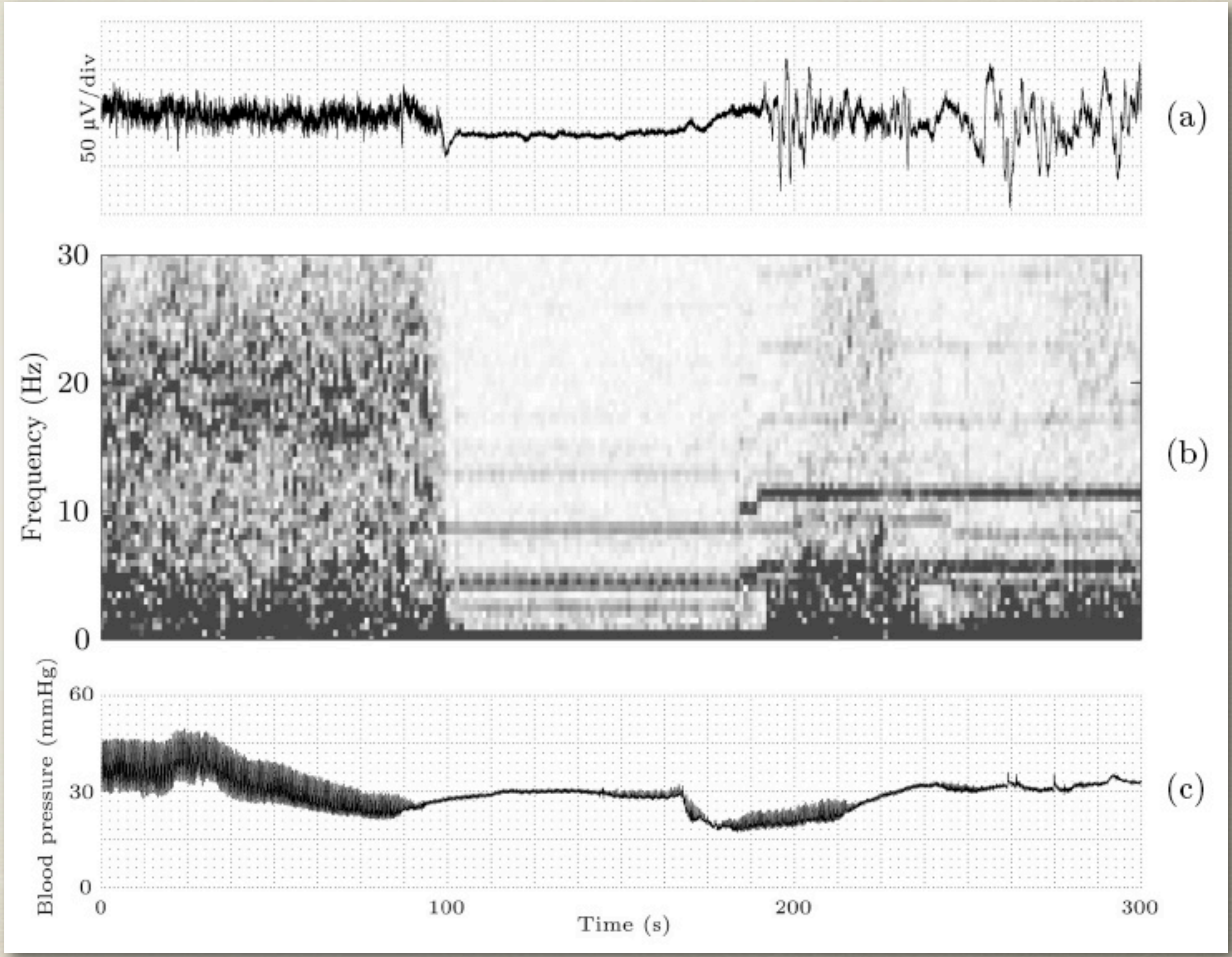


Heart Surgery of an Infant

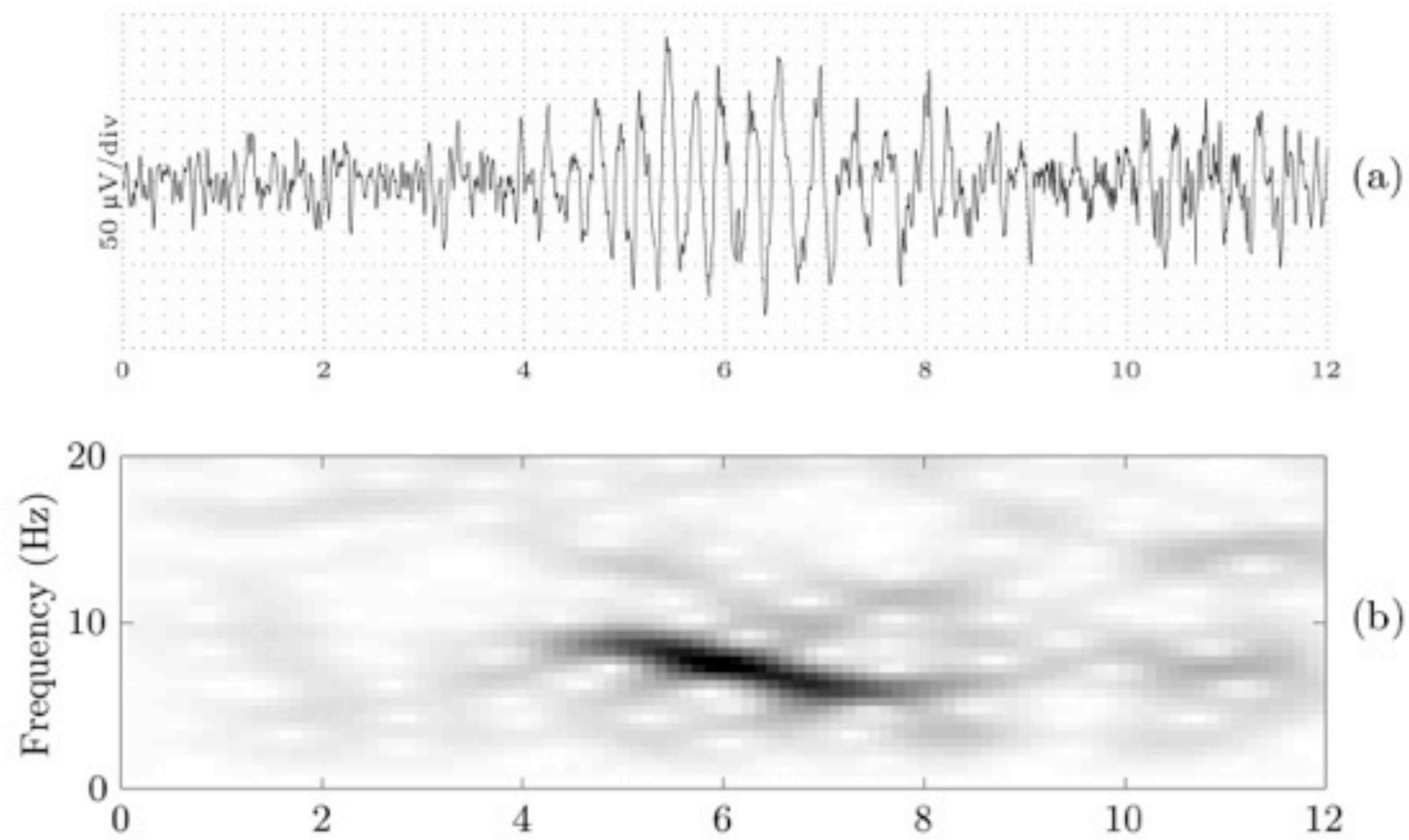
EEG

STFT

Blood pressure



EEG at the onset of an epileptic seizure



STFT using a 1-s window

STFT using a 2-s window

STFT using a 0.5-s window

STFT and Beyond

* The STFT (spectrogram) suffers from poor resolution in time and frequency.
* The use of a time-varying AR model may mitigate this problem, or
* the Wigner–Ville distribution (WVD), being a quadratic time-frequency distribution with better resolution in time and frequency...
* ...but with a problem – the presence of cross-terms – that must be remedied.

Time-Varying AR Model

we will outline how the parameters of a time-varying AR model,

$$x(n) = -a_1(n)x(n-1) - \cdots - a_p(n)x(n-p) + v(n), \tag{3.261}$$

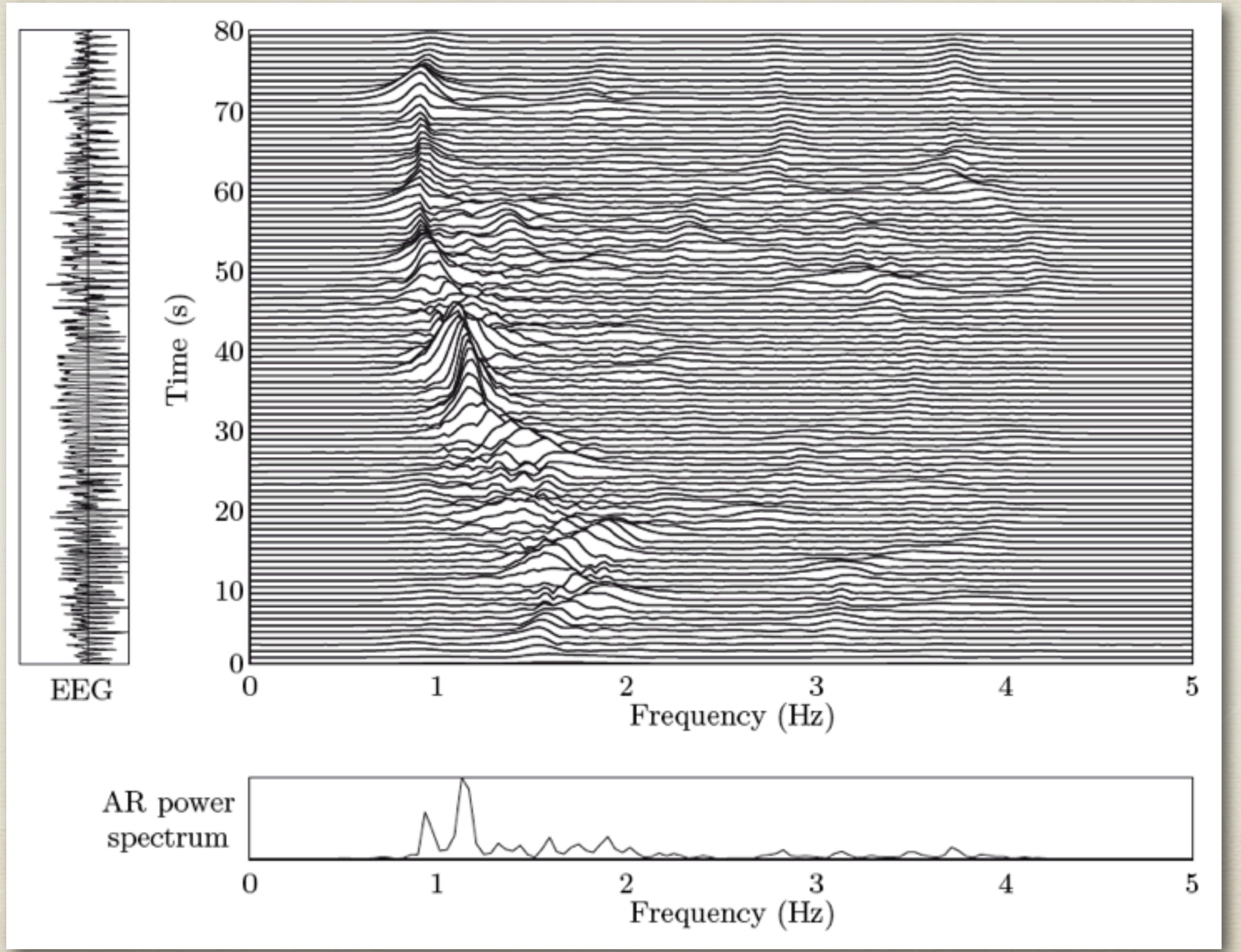
can be estimated under the assumption that temporal variations are relatively slow. In addition to the parameters $a_1(n), \ldots, a_p(n)$, the input noise $v(n)$ is also assumed to be time-varying with variance $\sigma_v^2(n)$. Hence, the time-varying power spectrum is given by

$$S_x(e^{\jmath\omega}, n) = \frac{\sigma_v^2(n)}{\left|1 + \sum_{k=1}^{p} a_k(n)e^{-\jmath\omega k}\right|^2}, \tag{3.262}$$

$$\mathbf{a}'_p(n+1) = \mathbf{a}'_p(n) - \frac{1}{2}\mu\nabla_{\mathbf{a}'_p} E\left[e_p^2(n)\right],$$

$$\mathbf{a}'_p(n+1) = \mathbf{a}'_p(n) - \mu e_p(n)\tilde{\mathbf{x}}'_p(n),$$

Time-varying AR during Seizure



WVD-based Time–Frequency Analysis

* Quadratic, nonparametric methods offer improved time–frequency resolution.
* The Wigner–Ville distribution (WVD) comes with a number of modifications introduced to address its limitations.
* The resulting methods do not involve any particular assumptions regarding the signal.
* A continuous-time framework is commonly adopted to facilitate the presentation.

The Ambiguity Function

The ambiguity function is designed to reflect uncertainty in both time and frequency associated with a signal x(t).

Two versions of x(t) are introduced, both shifted in time and frequency.

$$x(t;\nu,\tau) = x(t-\frac{\tau}{2})e^{-\jmath\nu t/2},$$
$$x(t;-\nu,-\tau) = x(t+\frac{\tau}{2})e^{\jmath\nu t/2}.$$

The ambiguity function is then defined as the correlation between the time and frequency shifted signal versions (one conjugated).

$$A_x(\tau,\nu) = \int_{-\infty}^{\infty} x^*(t;\nu,\tau)x(t;-\nu,-\tau)dt$$
$$= \int_{-\infty}^{\infty} x^*(t-\frac{\tau}{2})x(t+\frac{\tau}{2})e^{\jmath\nu t}dt.$$

Ambiguity function, con't

$$x(t) = s(t)\cos(\Omega_1 t) = x_1(t) + x_2(t), \tag{3.223}$$

where

$$x_1(t) = \frac{1}{2}s(t)e^{j\Omega_1 t}, \tag{3.224}$$

$$x_2(t) = \frac{1}{2}s(t)e^{-j\Omega_1 t}. \tag{3.225}$$

The corresponding ambiguity function becomes

$$\begin{aligned} A_x(\tau,\nu) &= \int_{-\infty}^{\infty}\left(x_1^*(t-\frac{\tau}{2}) + x_2^*(t-\frac{\tau}{2})\right)\left(x_1(t+\frac{\tau}{2}) + x_2(t+\frac{\tau}{2})\right)e^{j\nu t}dt \\ &= \frac{1}{2}A_s(\tau,\nu)\cos(\Omega_1\tau) + \underline{\frac{1}{4}A_s(\tau,\nu-2\Omega_1) + \frac{1}{4}A_s(\tau,\nu+2\Omega_1)}, \end{aligned} \tag{3.226}$$

where

$$A_s(\tau,\nu) = \int_{-\infty}^{\infty} s^*(t-\frac{\tau}{2})s(t+\frac{\tau}{2})e^{j\nu}dt. \tag{3.227}$$

The Analytical Signal

- It is obvious from the previous slide that the ambiguity function also includes two undesirable terms with identical shape but translated $\pm 2\Omega_1$.
- It is possible to remove such crosscorrelation without sacrificing signal information–the analytic signal.
- Since real-valued signals have symmetric frequency components, of which one is redundant, we only need to consider positive frequencies of the spectrum.

Analytical Signal in Math Terms

In the frequency domain, the analytic signal $x_A(t)$ of $x(t)$ is defined as:

$$X_A(\Omega) = \begin{cases} 2X(\Omega), & \Omega \geq 0; \\ 0, & \Omega < 0. \end{cases} \tag{3.228}$$

For $x(t) = s(t)\cos(\Omega_1 t)$, we have that

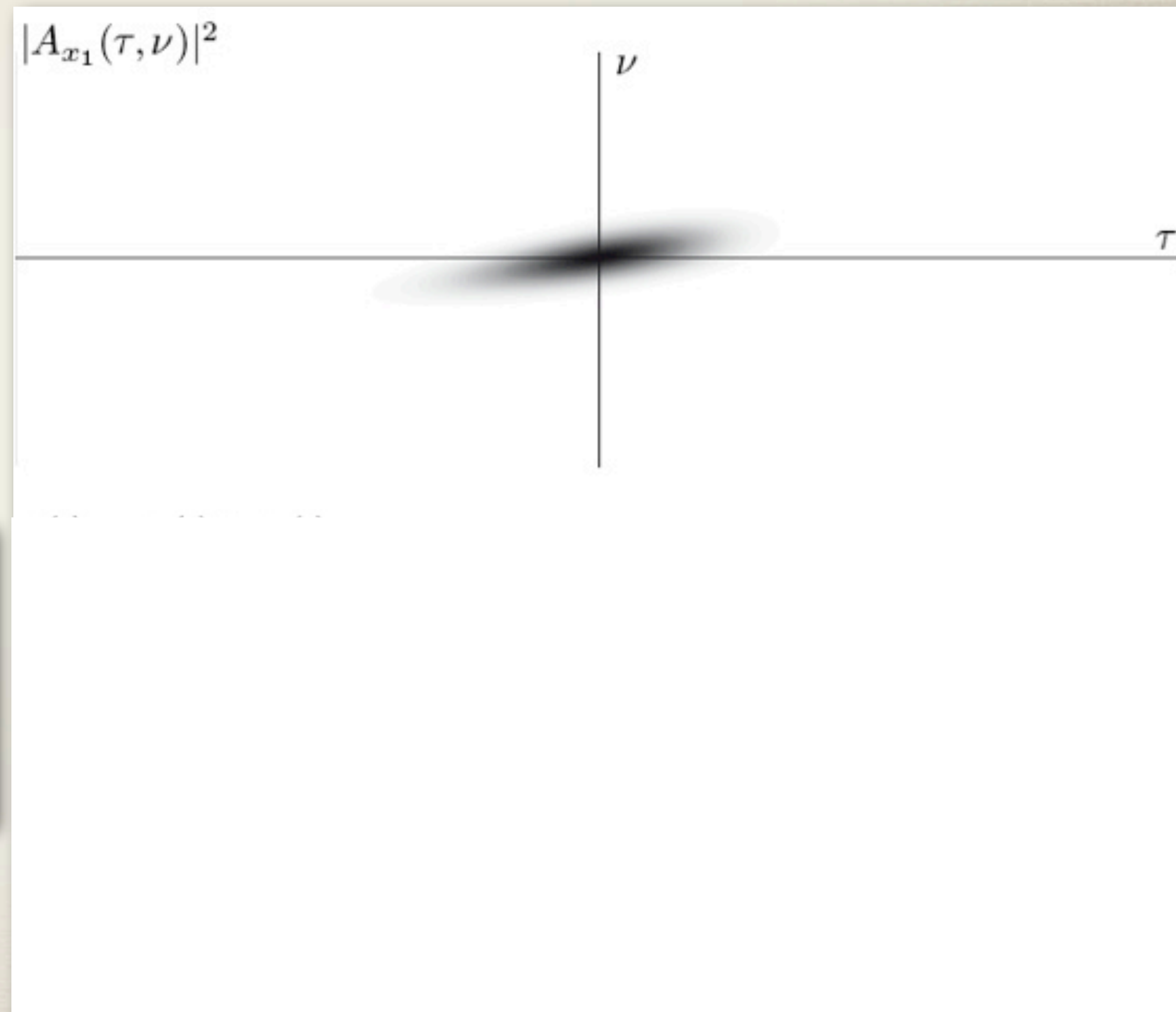
$$X_A(\Omega) = \begin{cases} S(\Omega - \Omega_1), & \Omega \geq 0; \\ 0, & \Omega < 0, \end{cases} \tag{3.229}$$

Using the analytic signal, the resulting ambiguity function no longer contains the two terms at $\pm 2\Omega_1$.

The analytic signal is always assumed in time-frequency analysis based on the WVD.

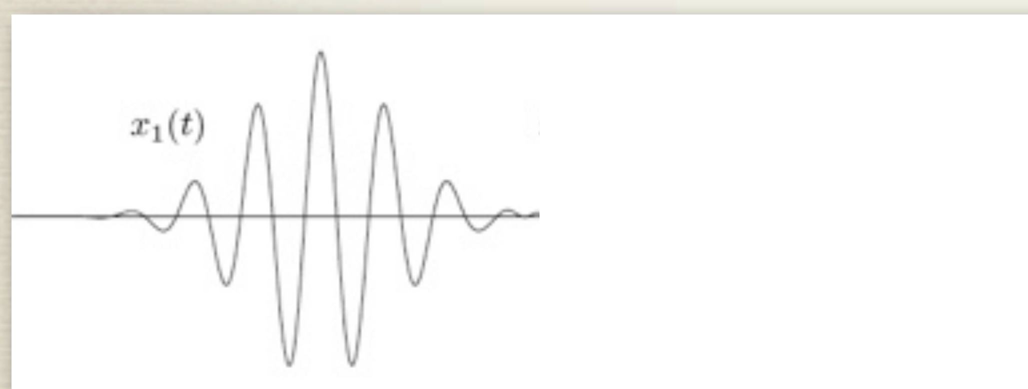
Ambiguity function, cont'

Ambiguity function



Analyzed signal

$$x_1(t) = e^{-(t-t_1)^2/\sigma} e^{j\Omega_1 t},$$



Note two frequencies

Wigner-Ville distribution (WVD)

The continuous-time definition of the WVD is given by the 2D-Fourier transform of the ambiguity function:

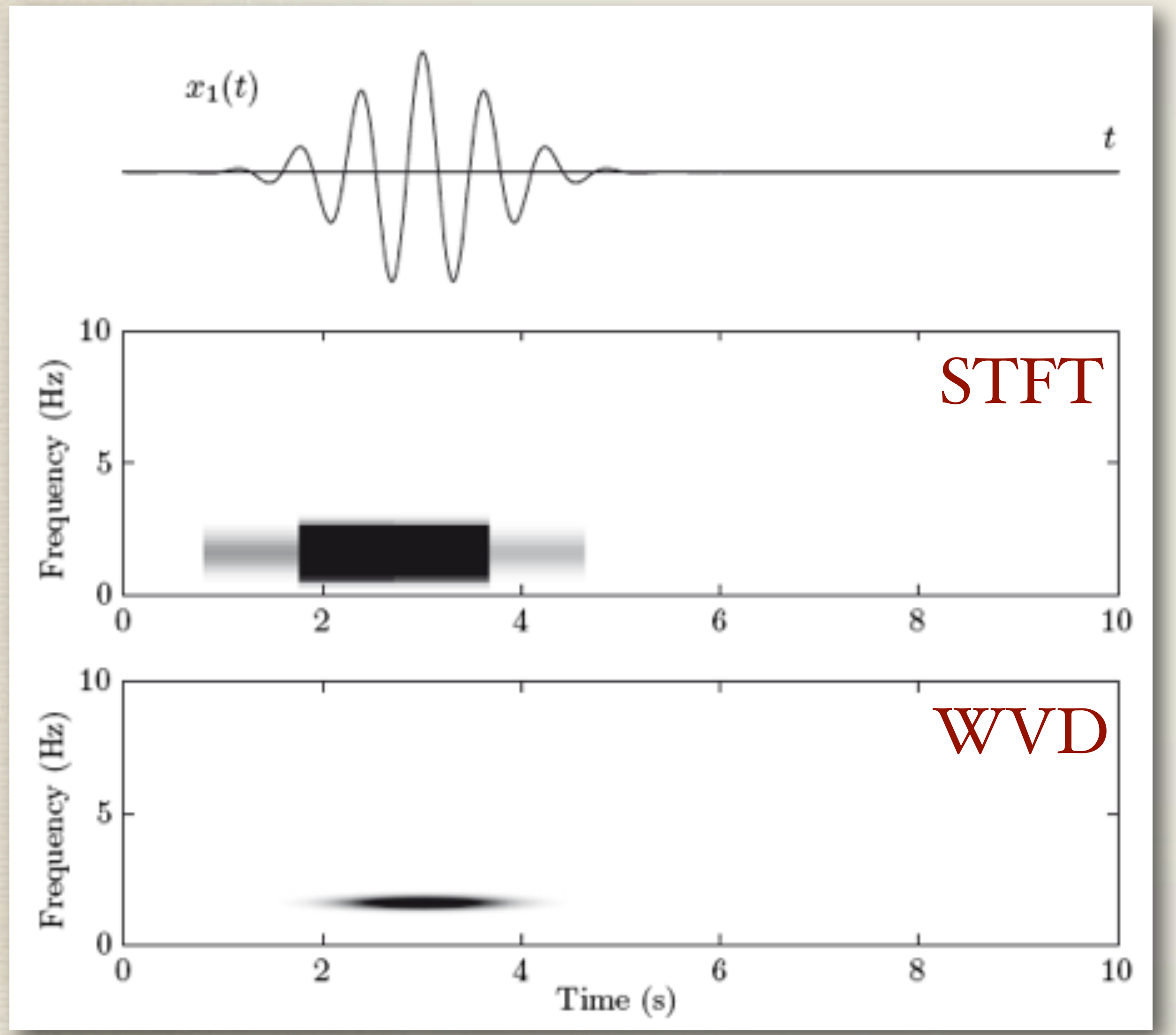
$$W_x(t,\Omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_x(\tau,\nu)e^{-j\nu t}e^{-j\Omega\tau}\,d\nu d\tau.$$

or expressed in terms of the signal itself:

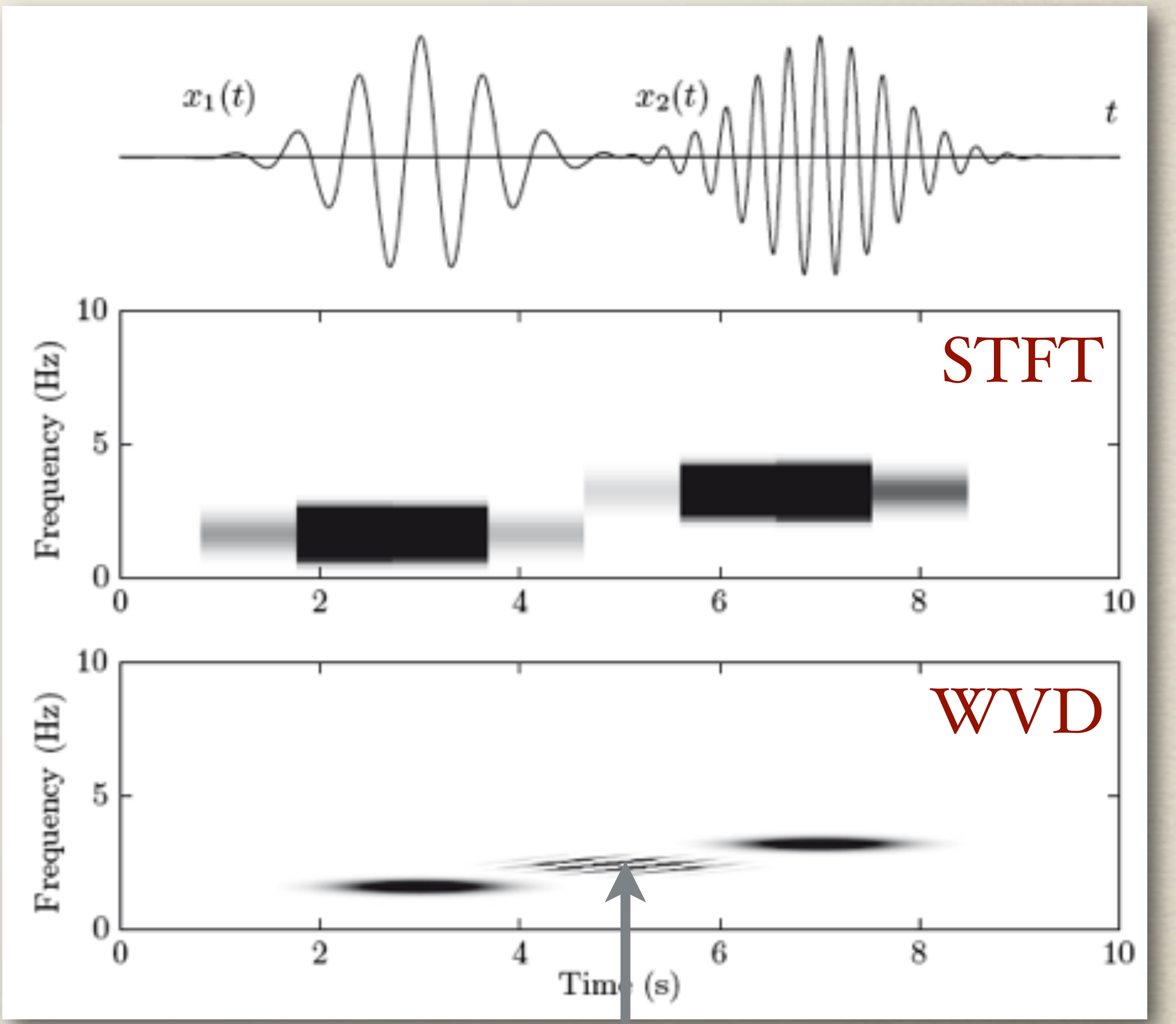
$$\begin{aligned} W_x(t,\Omega) &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^*(s-\frac{\tau}{2})x(s+\frac{\tau}{2})e^{-j\nu(t-s)}e^{-j\Omega\tau}\,ds d\nu d\tau \\ &= \int_{-\infty}^{\infty} x^*(t-\frac{\tau}{2})x(t+\frac{\tau}{2})e^{-j\Omega\tau}\,d\tau. \end{aligned} \tag{3.239}$$

Comparison of STFT and WVD

one-component signal



two-component signal



cross-term

The pseudo WVD

In order to emphasize the local properties in time of the analyzed signal, it is desirable to use the pseudo WVD, also known as the windowed WVD:

$$\breve{W}_x(t,\Omega) = \int_{-\infty}^{\infty} x^*(t-\frac{\tau}{2})x(t+\frac{\tau}{2})w(\tau)e^{-\jmath\Omega\tau}\,d\tau.$$

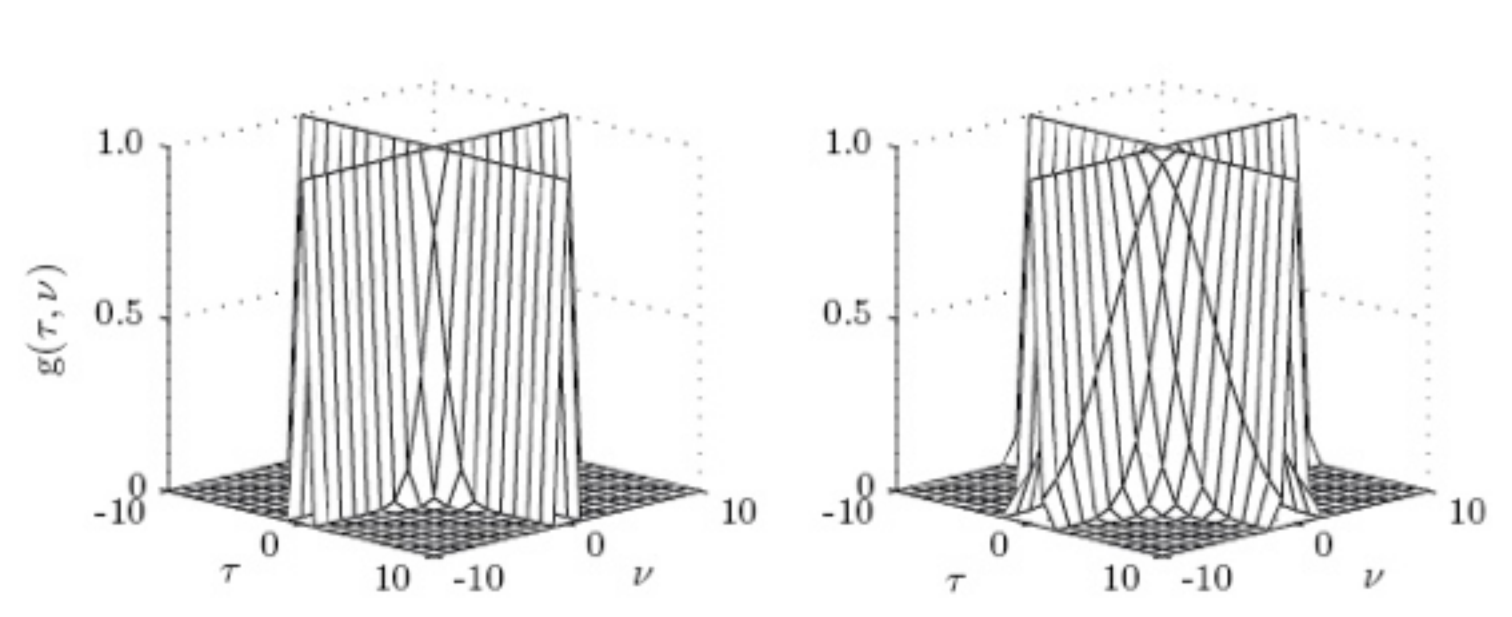
Cross-Term Reduction

A general class of time–frequency distributions
–Cohen's class–
has been introduced whose degrees of freedom can be exploited for mitigating the cross-term problem, defined by

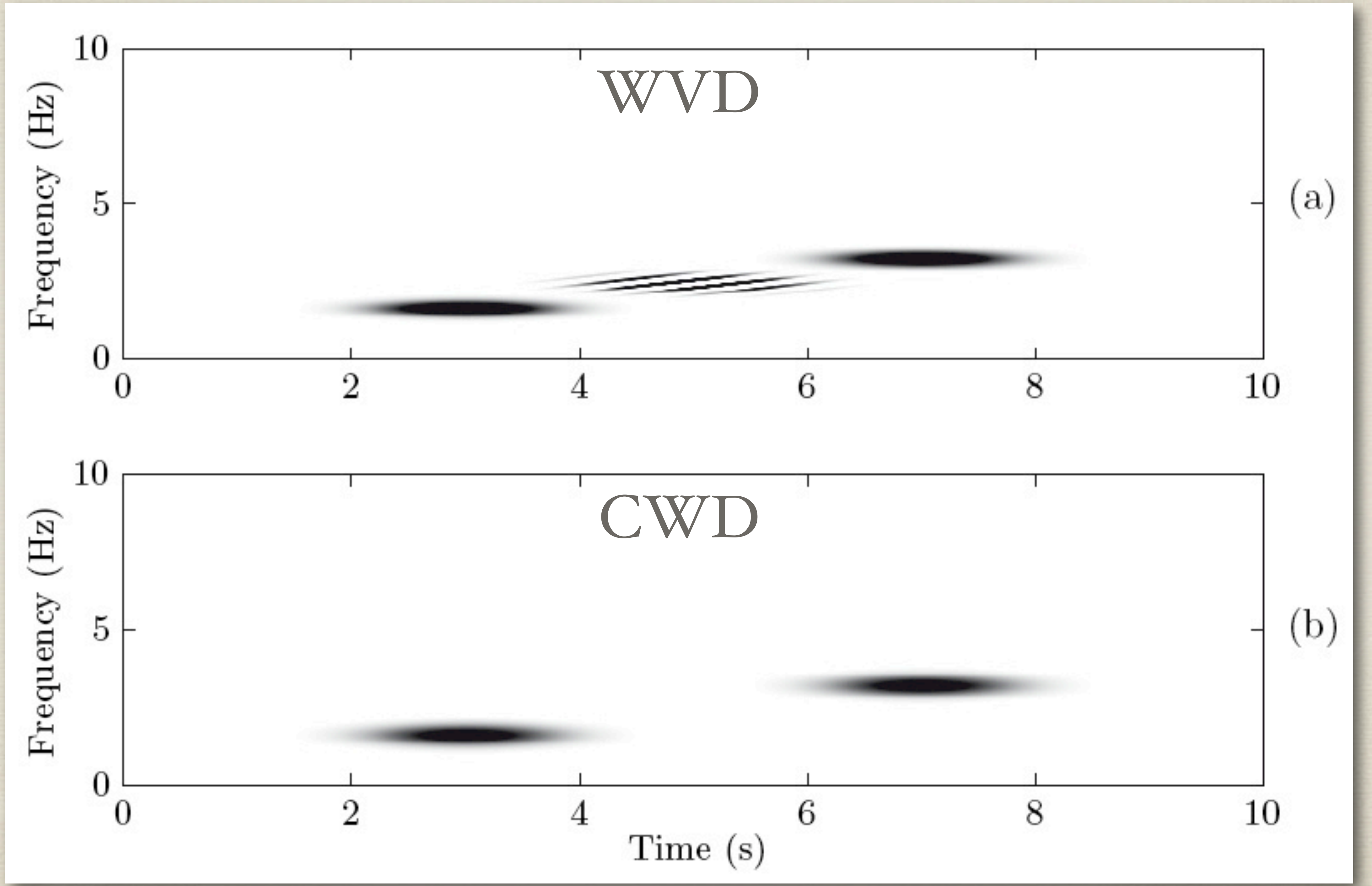
$$C_x(t,\Omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(\tau,\nu)A_x(\tau,\nu)e^{-\jmath\nu t}e^{-\jmath\Omega\tau}\,d\nu\, d\tau$$

The Choi–Williams distribution (CWD).

$$g(\tau,\nu) = e^{-\nu^2\tau^2/(4\pi^2\sigma)}, \quad \sigma > 0,$$



CWD and 2-Component Signal



EEG at the onset
of an epileptic seizure

STFT using a
window of 1 second

Wigner-Ville
distribution

Choi-Williams
distribution